Polar Plots and Circular Statistics

About the Author

Jeff Jenness is an independent GIS consultant specializing in developing analytical applications for a wide variety of topics, although he most enjoys ecological and wildlife-related projects. He spent 16 years as a wildlife biologist with the USFS Rocky Mountain Research Station in Flagstaff, Arizona, mostly working on Mexican spotted owl research. Since starting his consulting business in 2000, he has worked with universities, businesses and governmental agencies around the world, including a long-term contract with the United Nations Food and Agriculture Organization (FAO) for which he relocated to Rome, Italy for 3 months. His free ArcView tools have been downloaded from his website and the ESRI ArcScripts site over 190,000 times.

NAME: Polar Plots and Circular Statistics

Install File: PolarPlots.exe
Last modified: June 23, 2014

TOPICS: Polar plot, Rose plot, Circular Statistics, Angular Deviation, Angular Variance, Mean Resultant Length

AUTHOR: Jeff Jenness
    Wildlife Biologist, GIS Analyst
    Jenness Enterprises
    3020 N. Schevene Blvd.
    Flagstaff, AZ 86004   USA

    Email: jeffj@jennessent.com
    Web Site: http://www.jennessent.com)
    Phone: 1-928-607-4638

Description: This extension includes two tools. Both tools are available at the ArcView (aka ArcGIS Basic) license level.

1. The **Shapes to Segments** tool converts polygon or polyline features into polyline features of individual vertex-to-vertex segments, with options on various geometric attributes to calculate (starting and ending coordinates, length, azimuth).

2. The **Polar Plots** tool creates a graphic in the MXD layout illustrating the distribution of direction values in a polyline feature class (such as those generated in the **Shapes to Segments** tool) or table. The **Polar Plots** tool also has an option to calculate basic circular descriptive statistics on data.

Output: One tool produces polyline feature classes and the other tool produces a graphic plot in the MXD Layout.

Requires: ArcGIS 9.1 or better, at any license level.

Revision History on p. 45.

Recommended Citation Format: For those who wish to cite this extension, the author recommends something similar to:

Jenness, J. 2014. Polar Plots and Circular Statistics: Extension for ArcGIS. Jenness Enterprises. Available at: http://www.jennessent.com/arcgis/Polar_Plots.htm

Please let me know if you cite this extension in a publication (jeffj@jennessent.com). I will update the citation list to include any publications that I am told about.

Table of Contents

Installing the Polar Plots and Circular Statistics Extension

For ArcGIS 9.x

First close ArcGIS if it is open. Tools do not install properly if ArcGIS is running during the installation.

Install the *Polar Plots and Circular Statistics* extension by double-clicking on the file **PolarPlots.exe** and following the instructions. The installation routine will register the PolarPlots.dll with all the required ArcMap components.

The default install folder for the extension is named "Polar_Plots" and is located inside the folder "Program Files\Jennessent". This folder will also include some additional files and this document.

For ArcGIS 10.0

Note: This function will only work if you have ArcGIS 10 installed.

1. **First close ArcGIS if it is open.** The tools do not install properly if ArcGIS is running during the installation.

2. Install the *Polar Plots and Circular Statistics* extension by double-clicking on the file **PolarPlots.exe** and following the instructions. This installation routine will install the PolarPlots.dll and several ancillary files on your hard drive, but will **not** register the tool with ArcGIS.

3. Use Windows Explorer to open your installation folder. If you used the default values, then this folder will be located at "Program Files\ Jennessent\Polar_Plots\". This folder will also include some additional files and this manual.
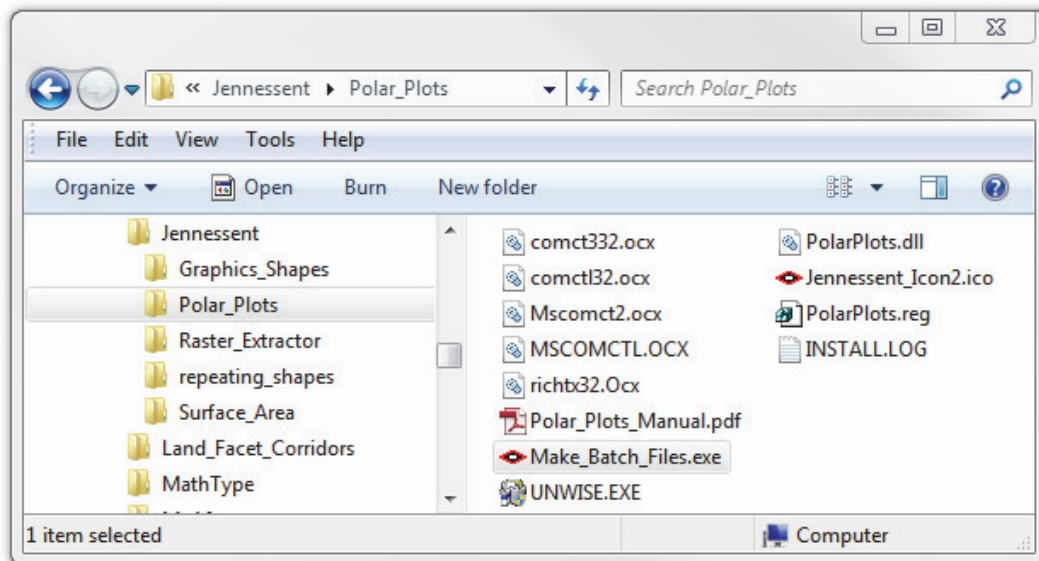
4. **For Windows XP:** Double-click the file "Make_Batch_Files.exe" to create registration and unregistration batch files that are properly formatted to your system.
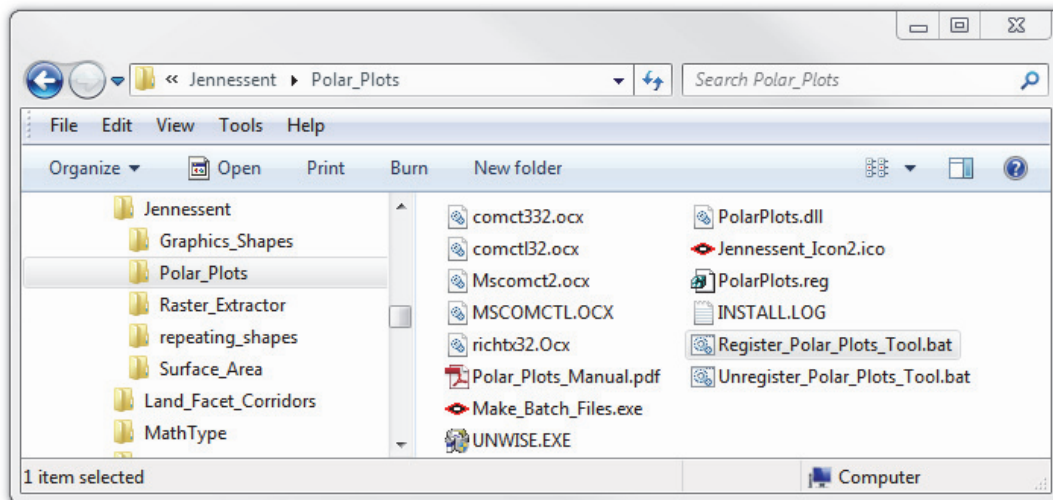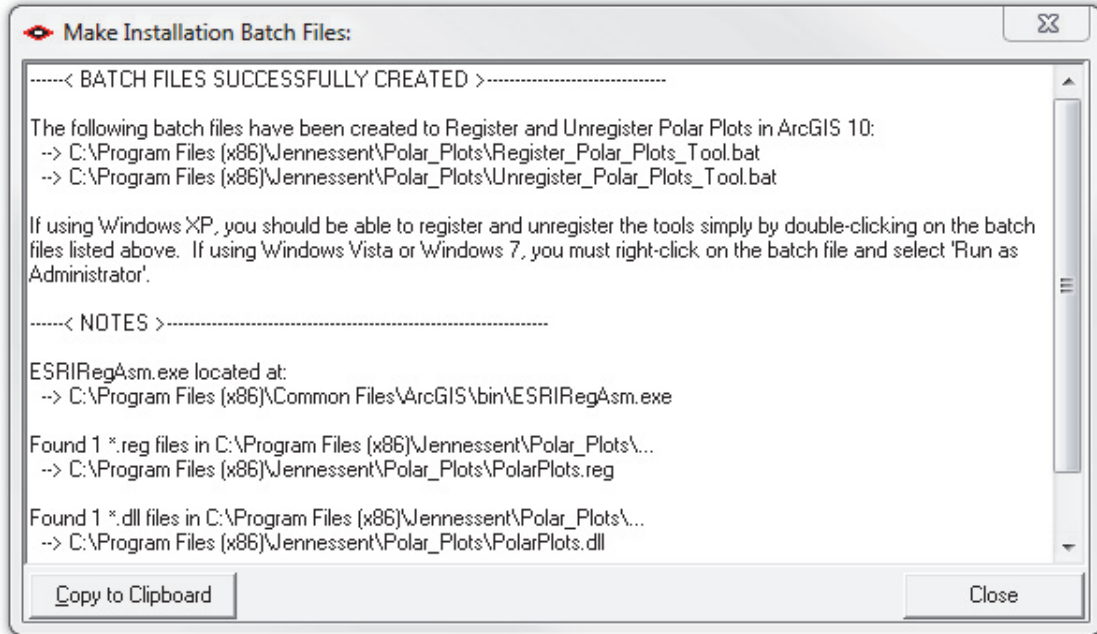
    a. Double-click the new batch file "Register_Polar_Plots_Tool.bat" to register the tool with ArcGIS 10.0.

    b. If the registration is successful, then you should see a "Registration Succeeded" notice.
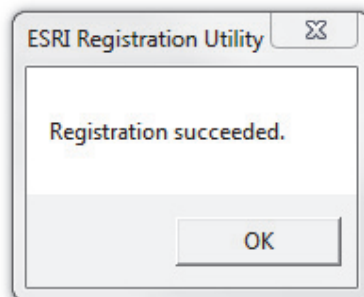
5. **For Windows Vista or Windows 7:** Right-click the file "Make_Batch_Files.exe", and then choose "Run as Administrator" to create registration and unregistration batch files that are properly formatted to your system.
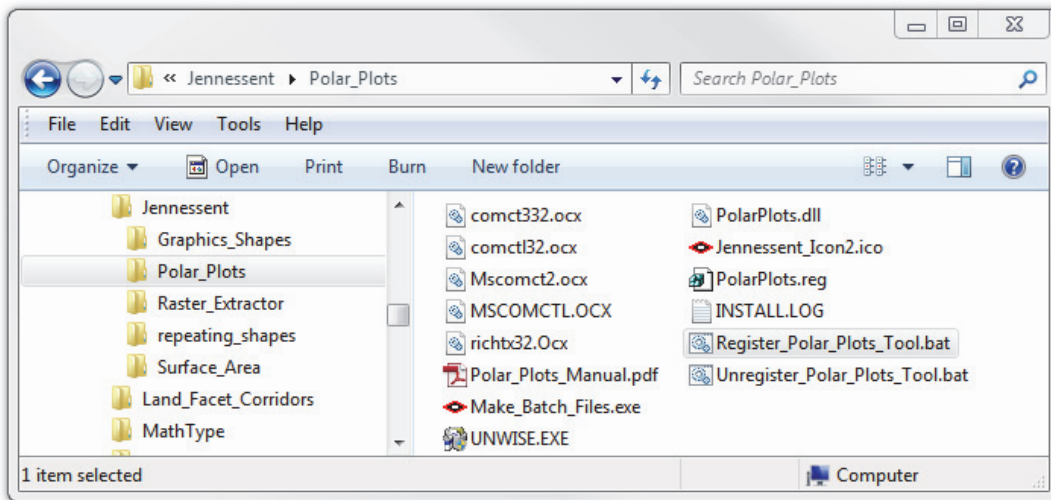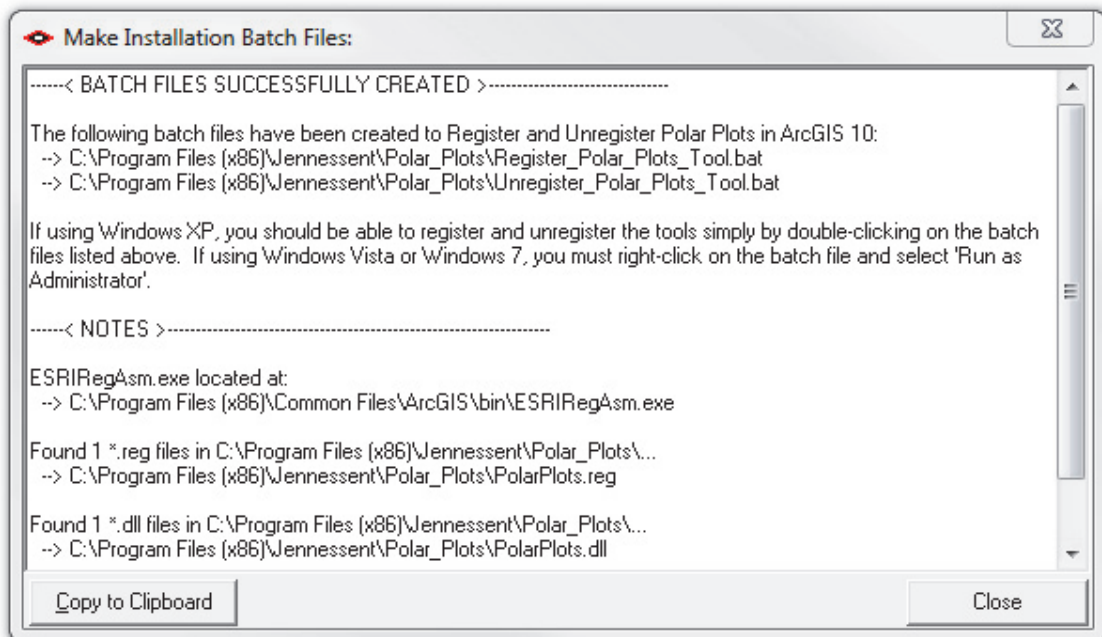
6. Right-click the new batch file "Register_Polar_Plots_Tool.bat", and then choose "Run as Administrator" to register all the tools with ArcGIS 10.0.

7. If the registration is successful, then you should see a "Registration Succeeded" notice.
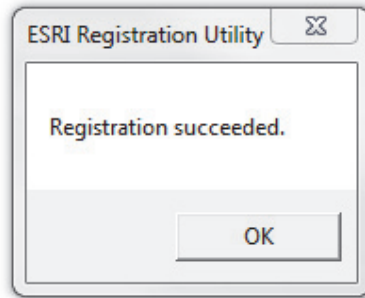
Note: For the concerned or curious, the batch file *Register_Polar_Plots_Tool.bat* contains a single line of text that looks similar to the following:

```
"C:\Program Files (x86)\Common Files\ArcGIS\bin\ESRIRegAsm.exe" /p:Desktop
"C:\Program Files (x86)\Jennessent\Polar_Plots\PolarPlots.dll" /f:"C:\Program
Files (x86)\Jennessent\Polar_Plots\PolarPlots.reg"
```

It directs the ESRI installer *ESRIRegAsm* to register the extension DLL *PolarPlots.dll* within ArcGIS, using GUID and Class ID values from the registry file *PolarPlots.reg* (also located in your installation directory). Both *Register_Polar_Plots_Tool.bat* and *PolarPlots.reg* may be opened and viewed using standard text editors such as Notepad or WordPad.

Viewing the Tools

This tool is installed as an extension in ArcMap, but it is a type of extension that is automatically loaded. You will not see this extension in the "Extensions" dialog available in the ArcGIS "Tools" menu. It is not dependent on any other extensions or any ArcGIS license level.

This extension does not include a separate toolbar so you will need to manually put the tools in an existing toolbar.

To do this, open your "Customize" tool by either:

1) Double-clicking on a blank part of the ArcMap toolbar, or

2) For ArcGIS 9, click the "Tools" menu, then "Customize", or

3) For ArcGIS 10, click the "Customize" menu, then "Customize Mode"

In the "Customize" dialog, click the "Commands" tab and scroll down to select "Jenness Enterprises":
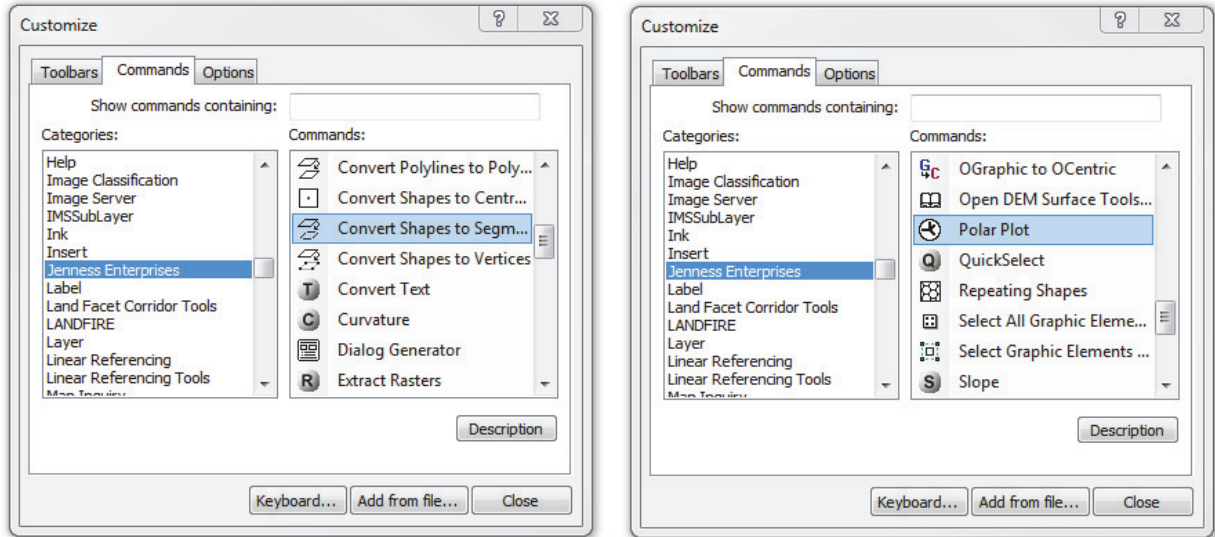
Finally, simply drag the **Convert Shapes to Segments** and/or the **Polar Plot** commands out of the Customize dialog up into any of the existing ArcGIS toolbars.

Uninstalling the Polar Plots and Circular Statistics Extension

For ArcGIS 9.x.

1) Close ArcGIS if it is open.

2) Click the Start button.

3) Open your Control Panel.

4) Double-click "Add or Remove Programs".

5) Scroll down to find and select "Polar Plots and Circular Statistics".

6) Click the "Remove" button and follow the directions.
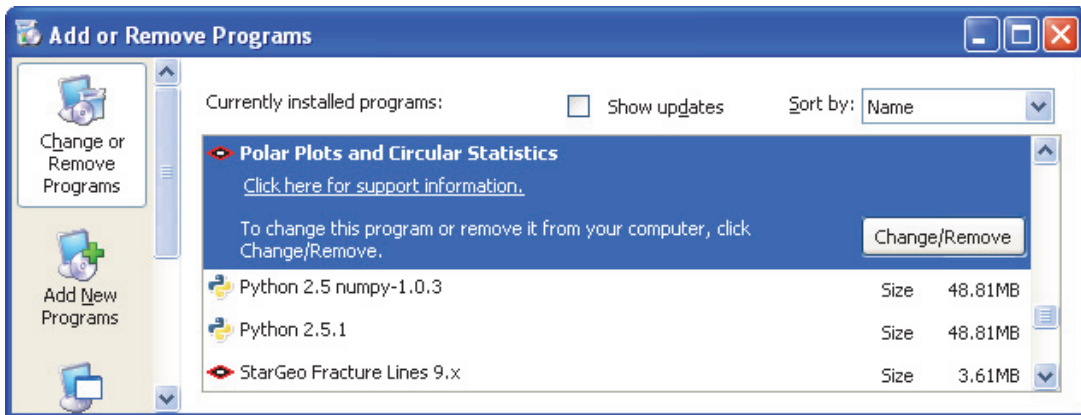
For ArcGIS 10.0

1) Close ArcGIS if it is open.

2) Use Windows Explorer to open your installation folder. If you used the default values, then this folder will be located at "Program Files\ Jennessent\Polar_Plots\". This folder will also include some additional files and this manual.
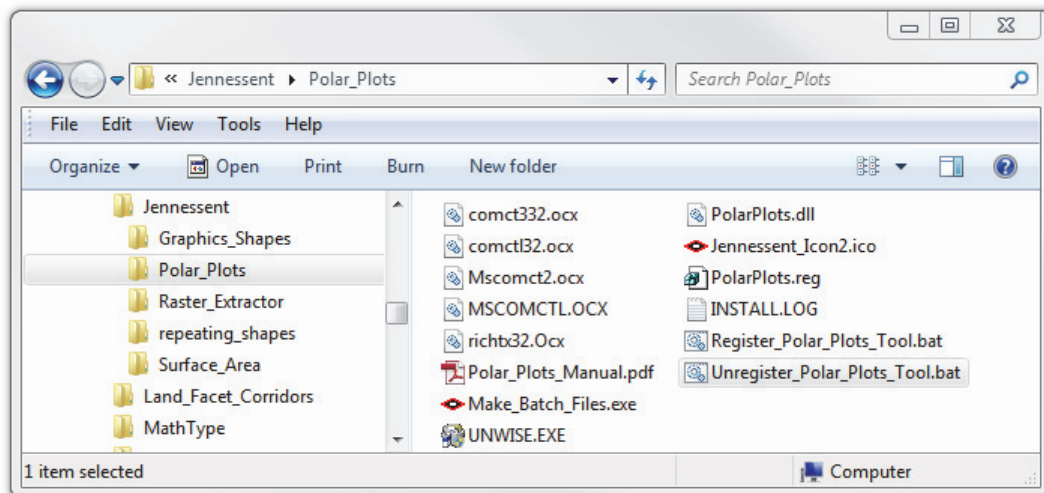
3) Find the file *Unregister_Polar_Plots_Tool.bat*. **IF YOU DO NOT SEE THIS FILE,** use the *Make_Batch_Files.exe* tool to create the batch file. Refer to Step 4 in "Installing

the Polar Plots and Circular Statistics Extension" above for instructions on how to use this tool.

4) **For Windows XP:** Double-click the file *Unregister_Polar_Plots_Tool.bat* to unregister the tool with ArcGIS 10.0.

   For Windows Vista and Windows 7: Right-click the file *Unregister_Polar_Plots_Tool.bat* and select "Run as Administrator" to unregister the tool with ArcGIS 10.0.
   If the unregistration is successful, then you should see an "Unregistration Succeeded" notice.
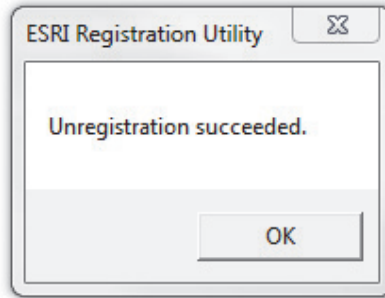
5) Click the Start button.

6) Open your Control Panel.

7) Double-click "Add or Remove Programs".

8) Scroll down to find and select "Polar Plots and Circular Statistics 10".

9) Click the "Uninstall" button and follow the directions.
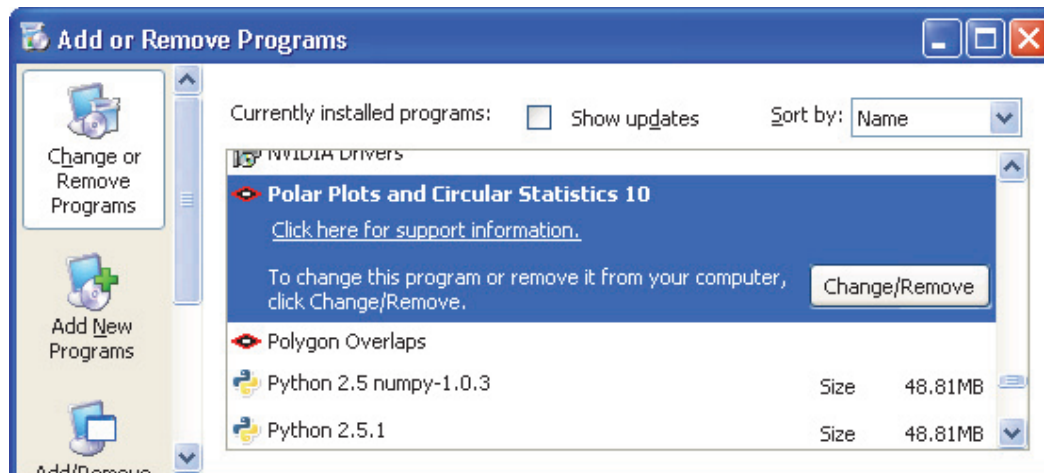
Note: For the concerned or curious, the batch file *Unregister_Polar_Plots_Tool.bat* contains something similar to the following single line of text:

```
"C:\Program Files\Common Files\ArcGIS\bin\ESRIRegAsm.exe" /p:Desktop /u
"C:\Program Files\Jennessent\Polar_Plots\PolarPlots.dll"
```

It directs the ESRI installer *ESRIRegAsm* to unregister the DLL *PolarPLots.dll* within ArcGIS. *Unregister_Polar_Plots_Tool.bat* may be opened and viewed using standard text editors such as Notepad or WordPad.

Troubleshooting

If Any of the Tools Crash

If a tool crashes, you should see a dialog that tells us what script crashed and where it crashed. I would appreciate it if you could copy the text in that dialog, or simply take screenshots of the dialog and email them to me at jeffj@jennessent.com. **Note:** Please make sure that the line numbers are visible in the screenshots! The line numbers are located on the far right side of the text. Use the scrollbar at the bottom of the dialog to make the line numbers visible.

"Object variable or With block variable not set" Error:

If you open ArcMap and immediately see the error dialog appear with one or more error messages stating that "Object variable or With block variable not set", then 90% of the time it is because ArcGIS was running when you installed the extension. The "Object" variable being referred to is the "Extension" object, and ArcGIS only sets that variable when it is initially opened.

The solution is usually to simply close ArcGIS and restart it. If that does not work, then:

1) Close ArcGIS

2) Reinstall the extension

3) Turn ArcGIS back on.

RICHTX32.OCX Error (also comct332.ocx, comdlg32.ocx, mscomct2.ocx, mscomctl.ocx, msstdfmt.dll errors):

If you see a line in the error dialog stating:

```
Component 'RICHTX32.OCX' or one of its dependencies not correctly registered: a file
is missing or invalid
```

Or if you see a similar error stating that one or more of the files *comct332.ocx, comdlg32.ocx, mscomct2.ocx, mscomctl.ocx* or *msstdfmt.dll* are missing or invalid, then simply follow the instructions for *RICHTX32.OCX* below, but substitute the appropriate file for *RICHTX32.OCX*.

This error is almost always due to the fact that new installations of Windows 7 and Windows Vista do not include a file that the extension expects to find. For example, the file "richtx32.ocx" is actually the "Rich Text Box" control that appears on some of the extension dialogs. The other OCX files refer to other common controls that might appear on the various extension dialogs.

The solution is to manually install the missing file (richtx32.ocx) yourself. Here is how to do it:

1) Open Windows Explorer and locate the file *richtx32.ocx* in your extension installation file.

2) If you are running a **32-bit version of Windows**, then copy *richtx32.ocx* to the directory

   ```
   C:\Windows\System32\
   ```

   If you are running a **64-bit version of Windows**, then copy *richtx32.ocx* to the directory

   ```
   C:\Windows\SysWOW64\
   ```

3) Open an "Elevated Command Prompt" window. This is the standard Windows Command Prompt window, but with administrative privileges enabled. You need these privileges enabled in order to register the OCX with Windows. **Note:** The Elevated Command Prompt opens up in the "..\windows\system32" directory, not the "..\Users\[User Name]" directory. The window title will also begin with the word "Administrator:"
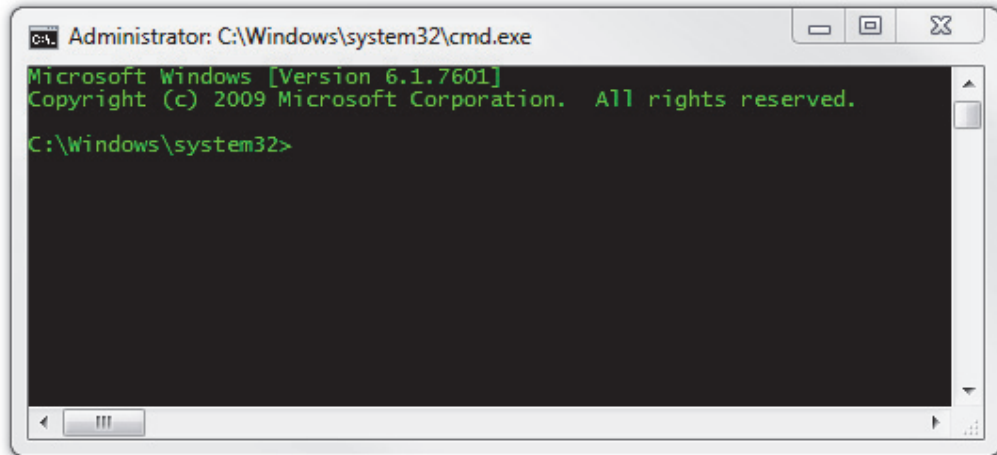
a. **Method 1:** Click the "Start" button, then "All Programs", then "Accessories" and then **right-click** on "Command Prompt" and select **Run as Administrator**.

b. **Method 2:** Click the "Start" button, and then click on the "Search Programs and Files" box. Type "cmd" and then click CONTROL+SHIFT+ENTER to open the Command window with Administrator privileges.
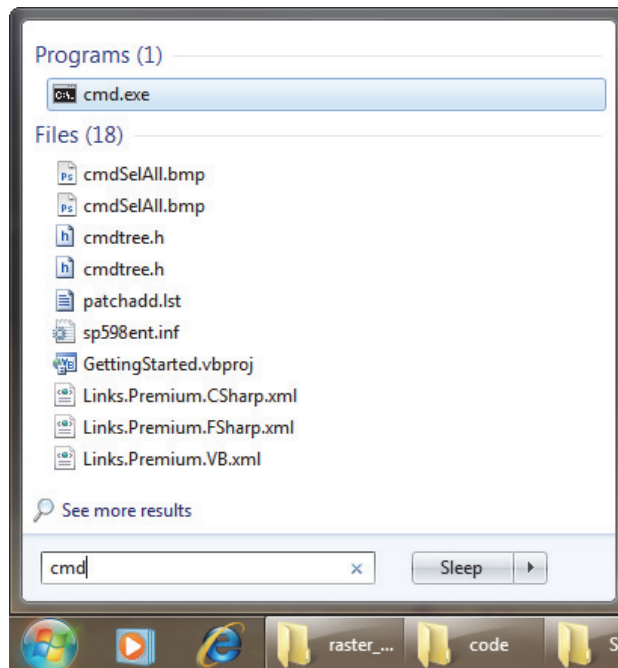
For more help on opening an Elevated Command Prompt, please refer to:

http://www.sevenforums.com/tutorials/783-elevated-command-prompt.html

http://www.winhelponline.com/articles/158/1/How-to-open-an-elevated-Command-Prompt-in-Windows-Vista.html

Or simply do a search for "Elevated Command Prompt".

4) Register the file *richtx32.ocx* using the Windows RegSvr function:

    a. If using a **32-bit version of Windows**, type the line

```
regsvr32.exe c:\windows\system32\richtx32.ocx
```
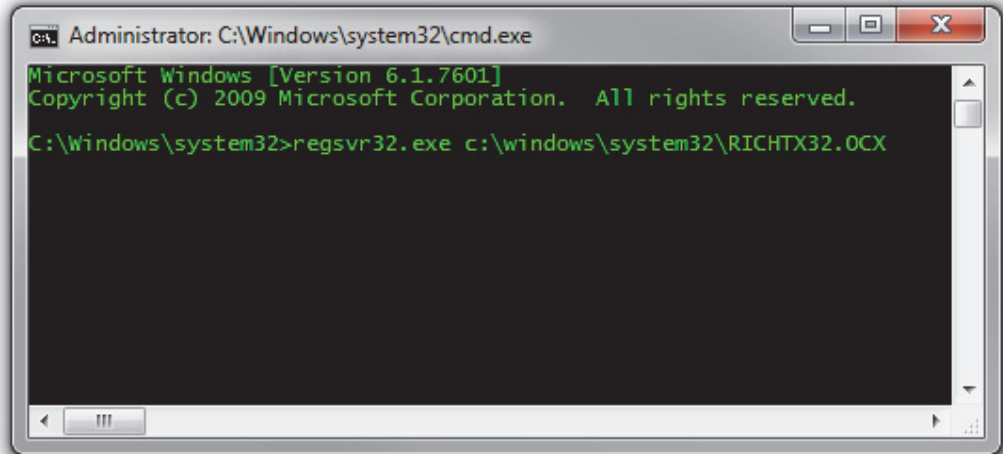

    b. If using a **64-bit version of Windows**, type the line

```
regsvr32.exe %windir%\syswow64\richtx32.ocx
```
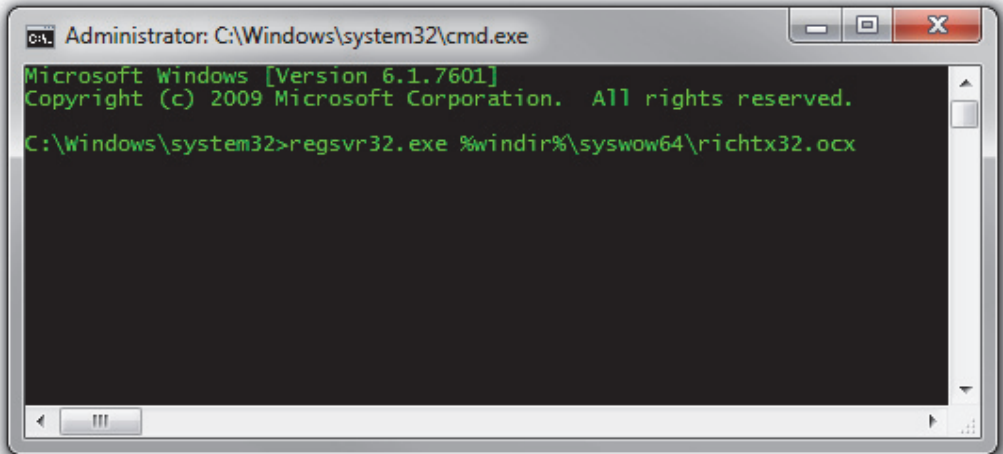

    c. Click [ENTER] and you should see a message that the registration succeeded.
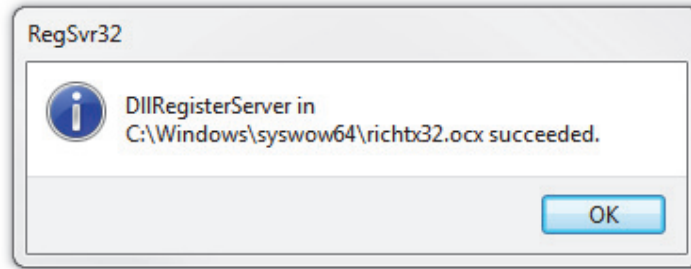
Issues with Directional Data

Many characteristics of wildlife habitat and behavior include a directional component. Animal movement may tend to angle toward or away some landscape feature, and the strength of the attraction or repulsion effect may change with proximity to that object. Aspect (i.e. the azimuth the landscape faces in the direction of steepest slope) is an intuitively valuable habitat variable because north-facing slopes are typically cooler and more mesic while south-facing slopes are generally warmer and more xeric (at least in the northern hemisphere!). Habitats can be dramatically different depending on which side of a hill you stand on.

Direction can be difficult to analyze statistically because of its circular nature. The difference between 1 degree and 360 degrees is the same as the difference between 1 degree and 2 degrees, making it difficult to plug into a typical statistical model. This article will discuss some issues we often face with directional data plus some basic ideas to describe and analyze it. Many of the concepts can also be applied to other periodic data such as time, but this article will focus on direction.

An additional complication is that sometimes we consider direction as a single value (the "forward" direction) and sometimes we count both the forward and reverse directions. In wildlife analysis, direction is usually treated as a single value. We are interested in where an animal is going and why it chooses to go in the direction it does. Or we are interested in the general aspect of a habitat block, and what that means to our species of interest. In some cases, however, we need to consider both the direction and its opposite value. Analysis of geologic fracture lines on the landscape should be considered to go in both directions. Analysis of trails or paths should be treated similarly, except in the rare case of one-way trails.

Azimuths and Geodesic Curves

In most cases the direction or azimuth value is easy to define. The aspect at a point can be derived with a little math from a DEM (see "Special Considerations for Aspect" on p. 26) or with a compass. The azimuth of a movement segment is simply the azimuth from the starting to the ending points of that segment.

Complications arise if we are interested in azimuths over long distances. The straight-line segment over a long distance is best approximated by a geodesic curve representing the great circle distance (i.e. "As the crow flies") between two points. The problem here is that, with a few rare exceptions, azimuth values change constantly over the course of a geodesic curve. Consider the geodesic curve connecting New York City, USA to Moscow, Russia:
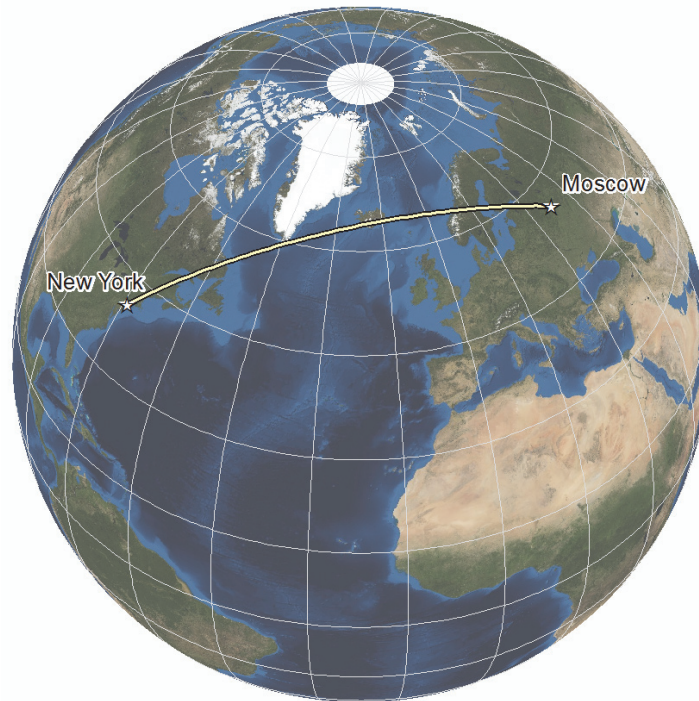
The line from New York starts at a bearing of 34°, continues in a straight line the entire way, but arrives in Moscow on a bearing of 130°.

For movement segments less than a few hundred miles, and sufficiently far away from the poles, the difference between starting and ending azimuths tends to be negligible. For long distances and for analysis near the poles, however, we should be aware of this phenomenon and be careful about how we define our directional values.

Graphically Representing Directional Data

Because of the circular nature of directional data, we typically use circular plots to display the distribution. Most of these circular plots are variations on the standard histogram, except wrapped in a circle so that the maximum possible X-axis value wraps back to touch the minimum possible value.

One of the more famous and historically interesting of these is the Rose Plot or Rose Diagram, named as such because the histogram bins resemble flower petals. This type of diagram was originally popularized by nurse and statistician Florence Nightingale, who used it to show seasonal patterns in soldier mortality (see http://en.wikipedia.org/wiki/Florence_Nightingale for the source of the famous image below). Nightingale actually called these diagrams "coxcombs" because they resemble the comb of a rooster.
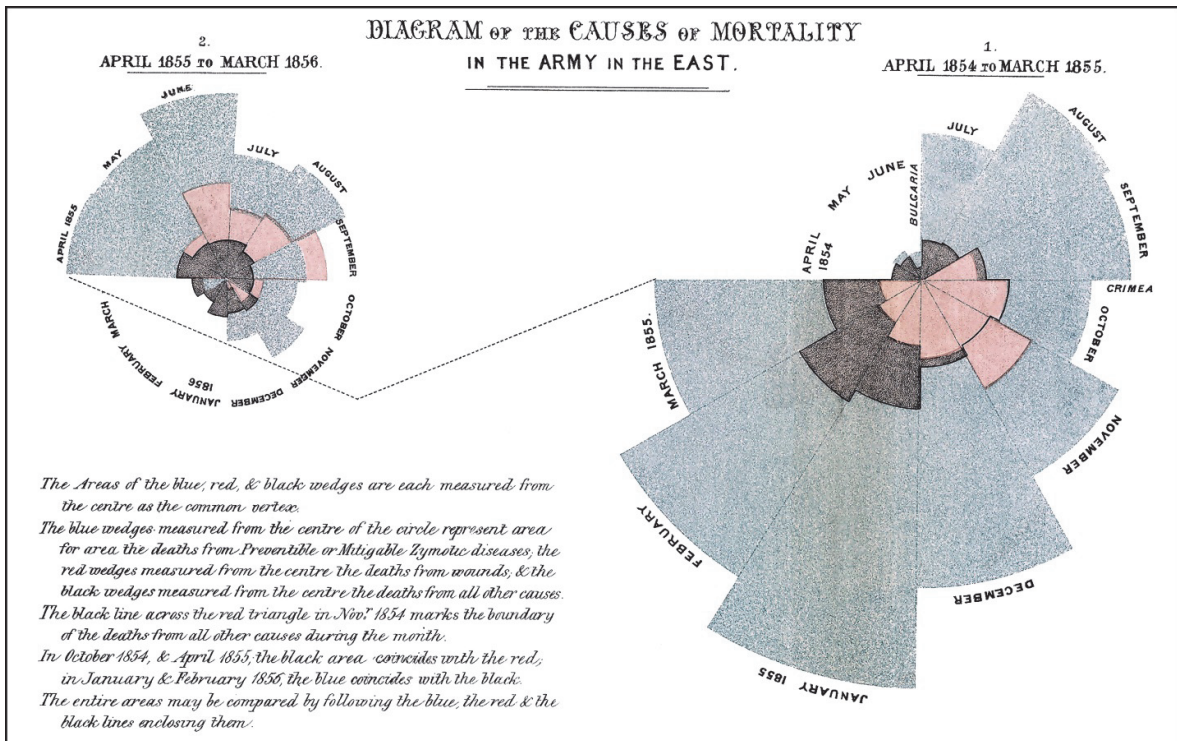
Nightingale's plots showed variation by season, but this method also works very well for displaying directional data. For example, given a set of trails in the San Francisco Bay area, we can use a rose plot to illustrate the general Northwest/Southeast orientation of those trails:
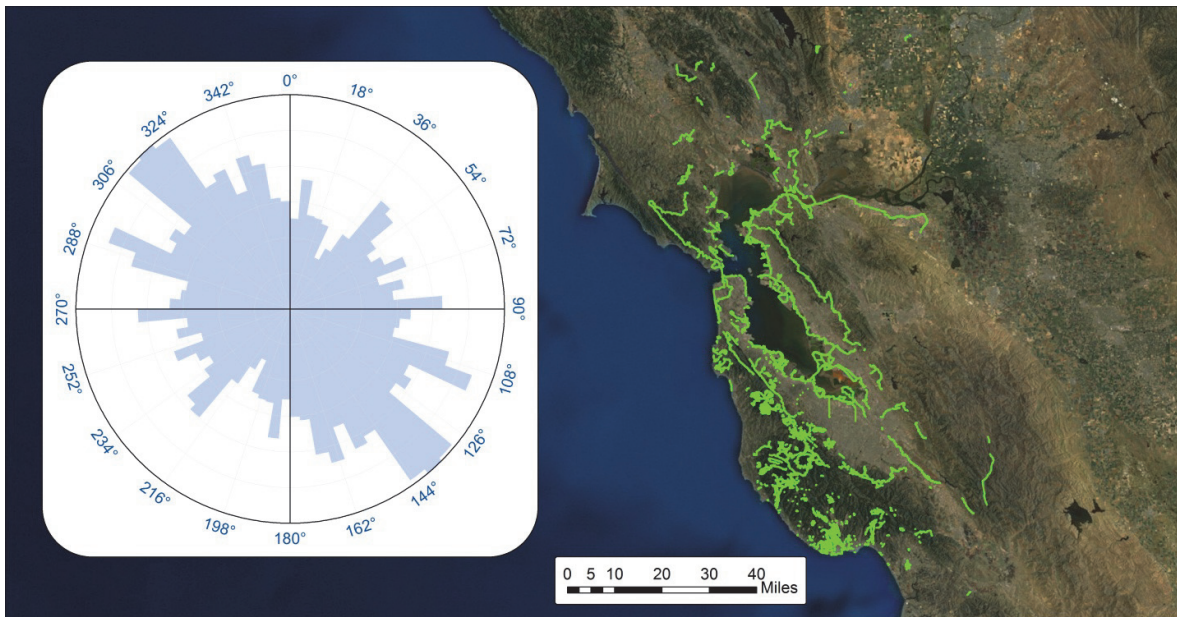
Common variations on the traditional rose plot include symbolizing the histogram bins as "Peaks and Valleys", and putting the bins on the outside of the circle rather than the inside.
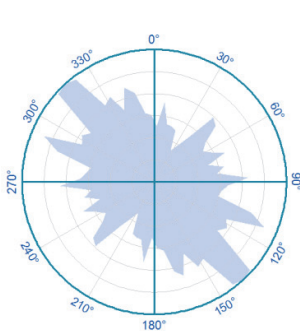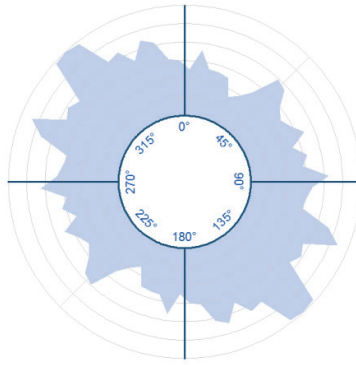
Peaks and Valleys   External Peaks and Valleys   External Histogram Bins
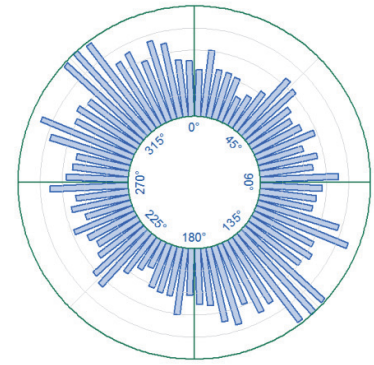
When only a few direction values need to be displayed, you can simply show the bearings in a circular plot.
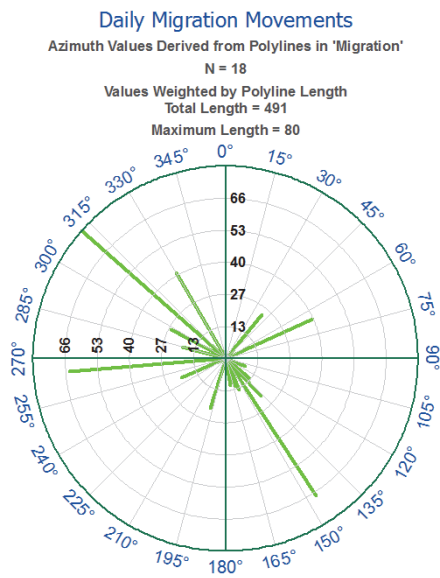
Another interesting method is a pie chart-type image where the "slices" are shaded according to the proportion of the data that faces that particular direction:

Roost Aspects
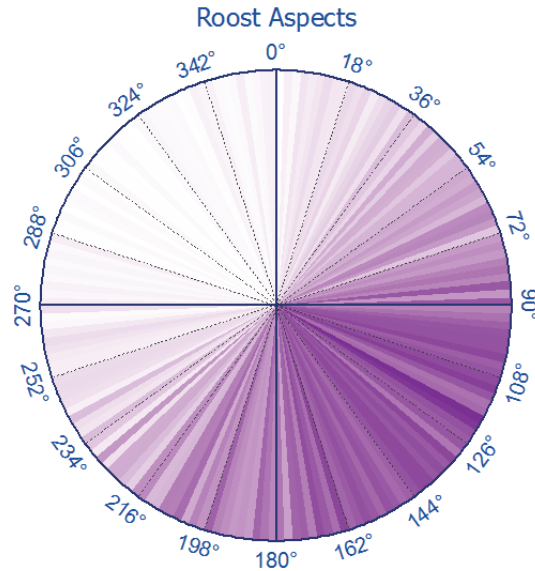
Although this last method is less effective at showing the shape of the distribution as other methods, it is still an interesting style of illustration because it is so visually intuitive. In the image above, it is easy to see that the observed roost locations predominately face toward the southeast. This extension provides methods to create all the plot types illustrated above (see p. 34 for a description of the tool). Additional software tools can be found with a Google search. Fisher (1995:15-30) also discusses alternative methods for graphing circular data.

Analyzing Directional Data: Circular Statistics

Before beginning this section, I want to emphasize two important points that people very often make mistakes with:

1) **Do not calculate the mean direction using the arithmetic mean!** This is especially frustrating because the arithmetic mean is sometimes correct and sometimes wildly incorrect. For example, the mean direction of 90° and 180° is 135°, which coincidentally is equal to the arithmetic mean $\left(\dfrac{90+180}{2} = 135 \right)$. But what is the mean direction of 359° and 1°? They are both pointing almost due north ($\pm$ 1°), and clearly the true mean direction is exactly due north. However, the arithmetic mean gives us 180° $\left(\dfrac{359+1}{2} = 180 \right)$, which is due *south* and exactly the opposite of the correct answer. The correct way to calculate the mean direction is described below.

2) **If you apply a Sine or Cosine transformation, make sure to convert the values to radians first!** Most analytical software and programming languages have sine and cosine functions, but these functions usually assume the values are in *radians*, not *degrees*. There are exactly 2π (~6.28) radians in a circle. This means that the software will assume that a difference of 3.14 (i.e. π) units is equivalent to going halfway around the circle. If your data are in degrees, then the software will interpret a change in 2° to be roughly equivalent to going a third of the way around the circle. Fortunately it is easy to convert to radians using the following formula:

Degrees to Radians:
$$Radians = \frac{Degrees \times \pi}{180} = \frac{Degrees \times (3.14159265358979)}{180}$$

20

It is possible that your software has a function that allows you to calculate sines and cosines from degrees (many calculators do), but even in this case you must remember to set the switch correctly.

Fortunately there are well-established methods available for analyzing circular or periodic data such as movement direction or aspect. The circular nature of the data lead to very specific and interesting approaches to calculating measures of central tendency and dispersion (see p. 43 for how to use this extension to calculate circular statistics). Some of the basic descriptive statistics are:

Mean Direction and Mean Resultant Length

Where: $\theta = $ Direction in Radians

$$C = \sum_{i=1}^{n} \cos\theta_i \qquad S = \sum_{i=1}^{n} \sin\theta_i \qquad R^2 = C^2 + S^2$$

Mean Direction: $\bar{\theta} = \begin{cases} \tan^{-1}(S/C) & S > 0, \ C > 0 \\ \tan^{-1}(S/C) + \pi & C < 0 \\ \tan^{-1}(S/C) + 2\pi & S < 0, \ C > 0 \\ \pi/2 & S > 0, \ C = 0 \\ -\pi/2 & S < 0, \ C = 0 \end{cases}$

Resultant Length: $R = \sqrt{R^2}$

Mean Resultant Length: $\bar{R} = \dfrac{R}{n}$

The equations for mean direction look a little confusing, but the logic is actually very intuitive. It is simply a process of vector addition, where each direction value is a single vector. Vector addition essentially connects all the direction vectors into a path, and the mean direction is just the direction to the last point on the path.

For example, consider a scenario with 4 direction values at 45°, 75°, 120° and 220°:
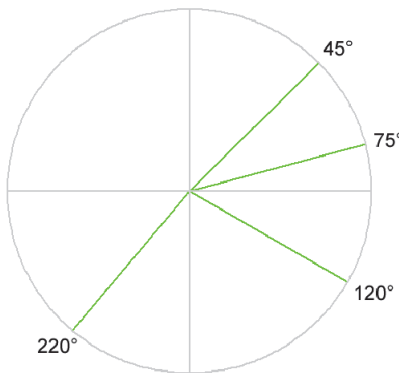
We connect the 4 bearings in a path (vector addition just adds up the ΔX and ΔY components of each vector, which is the same as treating each bearing as a segment in a path). It does not matter what order we connect the vectors in; they will always end up at the same point. The **Mean Direction** is the bearing from the start of the path to the end of the path.
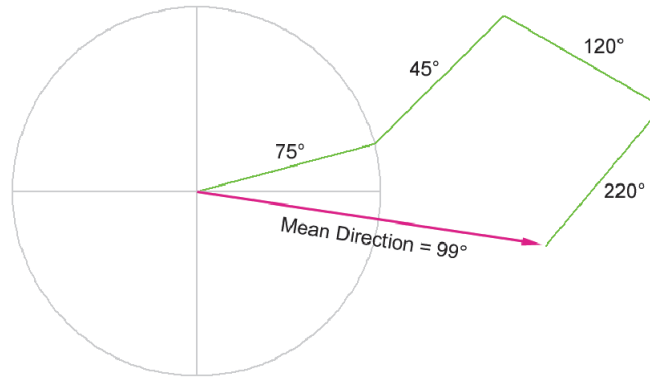
On a side note, this is also the way to calculate the mean direction of an actual observed movement path. If you have a series of locations from a GPS collar on an elk, for example, and you wonder what average direction the animal moved over the day, then that average direction is simply the direction from the first GPS location of the day to the last.

The **Mean Resultant Length** $\bar{R}$ is the basis for several values of dispersion (analogous to variance or standard deviation), and is calculated as the straight-line distance from the starting point to the ending point of the path divided by the number of segments (if the segments are unit vectors, where each vector has a length of 1) or by the total length of the path if the segments have variable lengths.
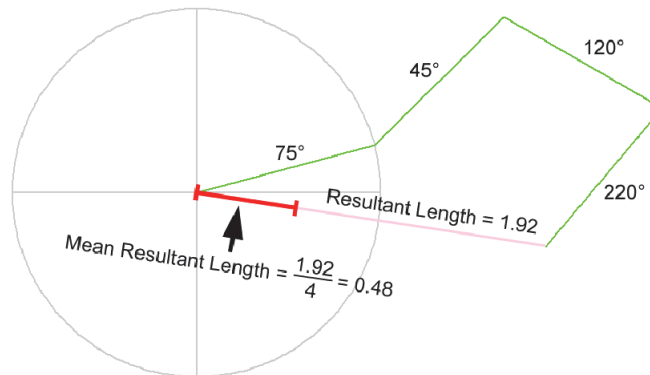
Notice that the mean resultant length has a potential range of 0 to 1. If all the vectors pointed in exactly the same direction, the *resultant length* would then be equal to the total path length and the *mean resultant length* would be equal to 1. This is the scenario with the minimum possible variance or dispersion in the vectors. The more the path wanders around, the shorter both the *resultant length* and the *mean resultant length* will be. If the path ended back at the origin, then both values would be equal to 0.

Variance and Standard Deviation:

Mardia and Jupp (2000), Fisher (1995) and Batschelet (1981) describe circular statistics analogous to variance and standard deviations, based on the mean resultant length $\bar{R}$.

Circular Variance: $V = 1 - \bar{R}$ (Mardia & Jupp, Fisher)

Angular Variance: $s^2 = 2\left(1 - \bar{R}\right)$ (Batschelet)

Circular Standard Deviation: $v = \sqrt{\left(-2\ln\bar{R}\right)}$ $\left(\text{In Radians}\right)$ (Mardia & Jupp, Fisher)

Angular Deviation: $s = \sqrt{s^2} = \sqrt{2\left(1 - \bar{R}\right)}$ $\left(\text{In Radians}\right)$(Batschelet)

The *Circular Standard Deviation* and *Angular Deviation* are both in units of Radians, but these can easily be converted to Degrees.

Radians to Degrees: $Degrees = \dfrac{180}{\pi}(Radians)$

Batchelet points out that *circular standard deviation* and the *angular deviation* tend to be close to each other for most values of $\bar{R}$. However, as $\bar{R}$ tends toward 0, *circular standard deviation* tends toward ∞ while *angular deviation* tends toward a maximum value of $\sqrt{2}$. Batchelet, citing Seyfarth and Barth (1972), presents a geometric derivation of angular deviation which also provides a visual sense of what the concept means.
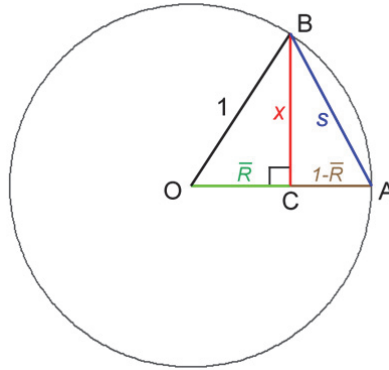
Given a unit circle with Radius = 1, and with:

- Point *A* defined as on the circle horizontal from the Origin *O*

- Point *C* defined as $\bar{R}$ distance along segment *OA* (remember that $\bar{R}$ will always be between 0 and 1)

- Point *B* defined as on the circle vertical from Point *C*

Then **s** (angular deviation) is just the length of the line connecting points A and B. The angular deviation can therefore be calculated by applying the Pythagorean theorem to $\triangle BCO$ and $\triangle ABC$:

From the Pythagorean Theorem on $\triangle BCO$:
$$x^2 + \bar{R}^2 = 1^2$$
$$\therefore x^2 = 1 - \bar{R}^2$$
From the Pythagorean Theorem on $\triangle ABC$:
$$s^2 = \left(1 - \bar{R}\right)^2 + x^2$$
Substituting for x^2 :
$$s^2 = \left(1 - \bar{R}\right)^2 + \left(1 - \bar{R}^2\right)$$
$$= \left(1 - \bar{R}\right)\left(1 - \bar{R}\right) + 1 - \bar{R}^2$$
$$= 1 - 2\bar{R} + \bar{R}^2 + 1 - \bar{R}^2$$
$$= 2 - 2\bar{R}$$
$$= 2\left(1 - \bar{R}\right)$$
$$\therefore s = \sqrt{2\left(1 - \bar{R}\right)}$$

Based on this illustration, it is easy to see that $s = 0$ when $\bar{R} = 1$, and that $s = \sqrt{2}$ when $\bar{R} = 0$

Note: A mean resultant length $(\bar{R})$ near 1 always implies a tightly focused set of directions, but a mean resultant length near 0 does **not** necessarily imply a high amount of variation or dispersion. All it implies is that the directions balance each other out. This can occur with a uniform distribution of directions, in which case there truly would be high dispersion. It can also occur, however, if you have a number of bearings in one direction and an equal number in the opposite direction. For example, perhaps you have a bird with distinct and separate roost and forage locations. Every day the bird travels to the forage location and then returns to the roost location. In this scenario, the bird only goes in two distinct and opposite directions (to the forage location, and then back to the roost location). The bird's movement directions are highly focused and predictable, but the mean resultant length would be 0 and consequently the variance would be high. As with most situations, plotting the distribution of the data is a good way to understand what is really happening.

Just as with standard statistics, there are a number of circular distributions and sophisticated analytical techniques available. These go beyond the scope of this newsletter article, but please refer to Jammalamadaka and SenGupta (2001), Mardia and Jupp (2000), Zar (1999; see especially ch. 26 and 27), Fisher (1995) and Batschelet (1981) for some good texts on circular statistics, distributions (i.e. the Fisher, Von Mises and Wrapped Normal distributions), circular hypothesis testing and other analytical techniques. There is also a good circular statistical package for R, originally based on Jammalamadaka and SenGupta's text. As of June, 2012, the manual for this package can be viewed at http://cran.wustl.edu/web/packages/circular/circular.pdf.

Other Methods to Analyze Directional Data

If we want to include direction as one of several independent predictor variables in a traditional statistical analysis, then we must transform the value into something approaching a linear measure. There are a few straightforward ways to do this, although any transformation will probably violate some of the assumptions of most traditional statistical tests.

Classification of Aspect Values

Probably the easiest transformation is to simply group your direction values into general and intuitive ranges (for example, "N" = 315 – 45, "E" = 45 – 135, "S" = 135 – 225, and "W" = 225 - 315), creating a categorical dataset which may be appropriate for some analyses.

Deviations from a Bearing

A simple and basic transformation is to convert your direction values into deviations from a direction of interest. For example, if you felt that the object of your study was likely to be affected by the north- vs. south-facing aspect phenomenon, then you might define your direction values in terms of "Deviation from North" where each value would reflect the distance, in degrees, from due North. Your full set of transformed values would range from 0 to 180 (see figure below). This option has the advantage of maintaining a constant interval between units, such that the difference in direction between 0 and 1 degree is the same as the difference between 90 and 91 degrees.

Sine and Cosine Transformations

Aspect values are often converted to sine and cosine values, essentially decomposing them into north-south and east-west components. *Remember to convert your direction values to radians before doing this transformation!* Sine values range from -1 (at due west) to 1 (at

due east), while cosine values range from -1 (at due south) to 1 (at due north). Note that this method does not maintain a constant interval between units. The sine and cosine values change by a variable amount depending on the direction, such that a change in sine corresponding to a change of 1 degree = 0.00015 when going from 90 to 91 degrees, but increases by more than 2 orders of magnitude to 0.017 when going from 180 to 181 degrees. This issue may be important in your statistical analysis if your method assumes that your data are interval-level.
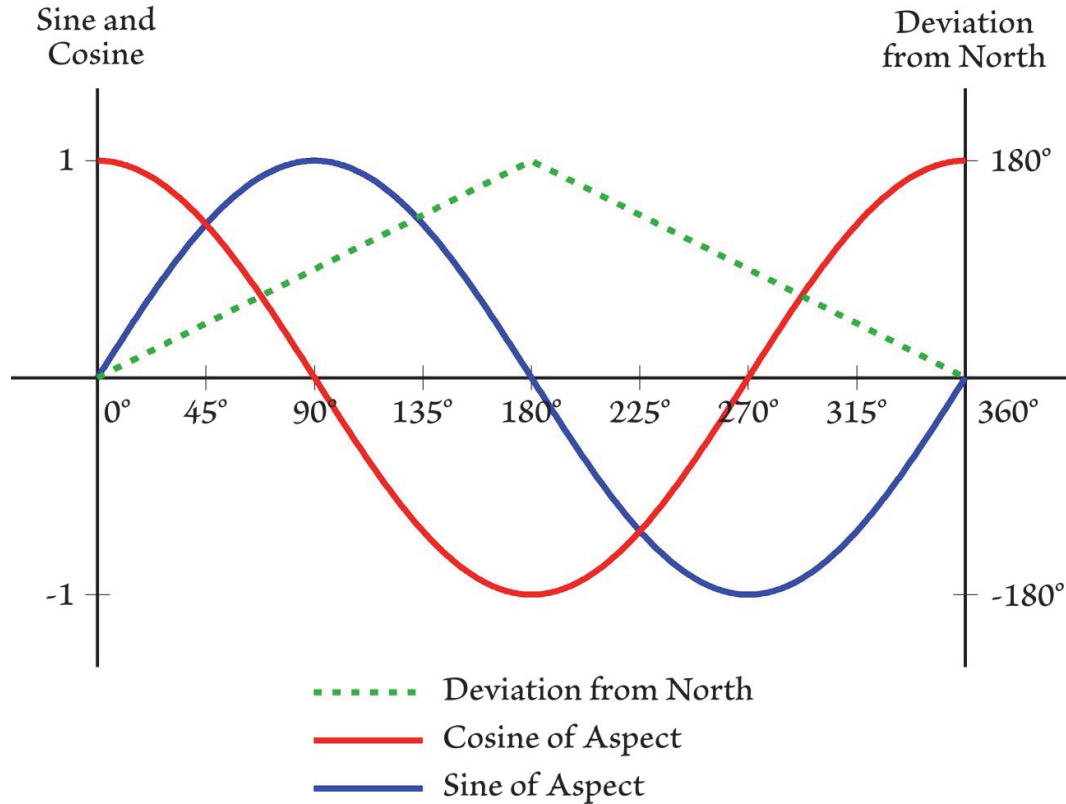
Trimble and Weitzman (1956) and Beers et al. (1966) suggest an interesting alternative combining two of the approaches above, rescaling aspect values based on an optimum bearing (135° for Trimble & Weitzman, and a general equation for any aspect of interest for Beers et al.) then taking the sine of the rescaled values, then adding 1, which they put to extensive use in site productivity research for timber stands. Beers et al.'s general equation is:

A' = Transformed Aspect Code, in Degrees

$$= \sin\left[A + \left(90° - A_{Max}\right)\right] + 1$$

$$= \cos\left(A_{Max} - A\right) + 1$$

Where:

A_{Max} = The aspect which is to be assigned the highest numerical value on the transform scale

Note: Beers et al.'s equation presumes that you can calculate the sine or cosine of a value in *degrees*. Many calculators can do this, but many programming languages require your values to be in *radians.* If you need to convert your values, the Degrees-to-Radians conversion formula is on p. 20 of this manual.

Special Considerations for Aspect

How Aspect is Calculated

Aspect is calculated from the directional East-West and North-South gradients at a particular point. There are various methods for estimating these directional gradients from a 3x3 set of elevation points, but in general aspect is defined as the direction of maximum slope. Given that G = east-to-west gradient and H = north-to-south gradient, then aspect is calculated as follows:

$$\text{Mathematical Direction (in degrees)} = \text{atan}2(H, -G)\left(\frac{180}{\pi}\right)$$

The "atan2" function is a method of calculating the arctangent $\left(\tan^{-1}\right)$ using two values (ΔX and ΔY), and it has a possible range representing a full circle (-π to +π radians, or -180° to +180°). The standard arctangent function can only produce values ranging from $-\frac{\pi}{2}$ to $+\frac{\pi}{2}$, or -90° to +90°. This atan2 function is the same one used to calculate mean direction above, and is calculated as follows:

$$\text{atan}2(y, x): \quad \bar{\theta} = \begin{cases} \tan^{-1}\left(\dfrac{y}{x}\right) & y > 0, \ x > 0 \\[2mm] \tan^{-1}\left(\dfrac{y}{x}\right) + \pi & y \geq 0, \ x < 0 \\[2mm] \tan^{-1}\left(\dfrac{y}{x}\right) - \pi & y < 0, \ x < 0 \\[2mm] \pi/2 & y > 0, \ x = 0 \\[2mm] -\pi/2 & y < 0, \ x = 0 \\[2mm] \text{Undefined} & y = 0, \ x = 0 \quad (\text{i.e. no movement}) \end{cases}$$

Mathematical direction is different than compass direction. In the mathematical polar coordinate system, 0° is equivalent to due east and polar directions increase in a counter-clockwise direction. Compass direction starts at north and increases in a clockwise direction. Therefore we must convert the mathematical direction to a compass aspect according to the following rules:

if *Mathematical Direction* > 90° then *Aspect* = (450 – Direction)

Otherwise *Aspect* = (90 – Direction)

ArcGIS uses Horn's method (Horn 1981, Burrough and McDonnell 1998)to calculate the east-west (G) and north-south (H) gradients from a 3x3 array of elevation points. The figure below shows the 8 elevation values that are used to calculate aspect at the central cell XX (note that the formula does not use the elevation value at the central cell):

```
Z1 ---- Z2 ---- Z3
 |       |       |
 |       |       |
 |       |       |
Z4 ---- XX ---- Z6
 |       |       |
 |       |       |
 |       |       |
Z7 ---- Z8 ---- Z9
```
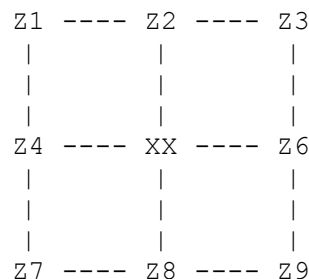
In Horn's Method, the north-south gradient is calculated from all three columns in the 3x3 array, with the central column weighted twice as high as the two outer columns. The east-west gradient is calculated in a similar manner.

$$\text{East-West Gradient} = G = \frac{\left[(Z_3 + 2Z_6 + Z_9) - (Z_1 + 2Z_4 + Z_7)\right]}{8\Delta x}$$

$$\text{North-South Gradient} = H = \frac{\left[(Z_1 + 2Z_2 + Z_3) - (Z_7 + 2Z_8 + Z_9)\right]}{8\Delta y}$$
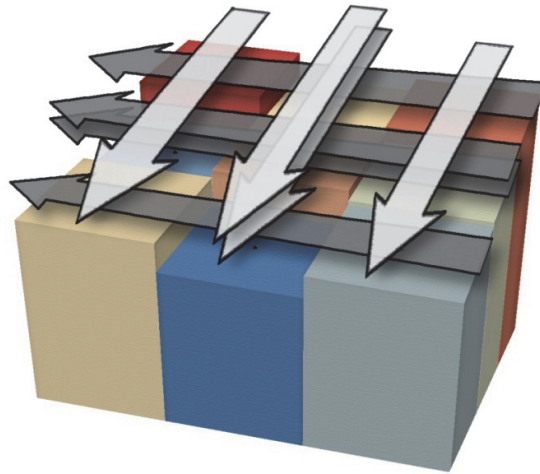
Is this the best method to calculate aspect?

Interestingly, Jones (1998) compared several slope and aspect algorithms on a mathematical surface in which the true exact slope and aspect at any point could be calculated. He found that Horn's method did better than most methods, but was marginally worse than a method called the 4-cell method. He then tested all methods on a true landscape DEM and found that the rankings among methods were consistent, so Horn's method still came in 2nd place.

Another tidbit: Michael Hodgson (1995) published a paper using the same mathematical surface that Jones (1998) did, in which he demonstrates that the area represented by computed slope and aspect is actually larger than the raster DEM cell size. 8-cell algorithms (such as *Horn's* method) more accurately reflect values for an area approximately twice the cell size, while 4-cell algorithms (such as the *4-Cell* method) reflect an area approximately 1.6X the original cell area.

In sum, a method called the 4-cell method is simpler, slightly quicker to calculate, more precise and marginally more accurate than Horn's method.

For those who are interested, the author offers a free ArcGIS extension that will calculate slope and aspect using a variety of methods including both Horn's method and the 4-cell method (see http://www.jennessent.com/arcgis/surface_area.htm). This extension does not require Spatial Analyst.

Issues with Aspect

As a consequence of determining aspect from East-West and North-South components, aspect can be difficult to calculate from geographically-projected (latitude/longitude) data. The equations above all work for 3x3 arrays in which the elevation locations are all equally spaced along both the X- and Y-axes. However, this is not the case with latitude / longitude data. Raster cells that are square in unprojected space actually form trapezoids when viewed on the actual surface of the planet.
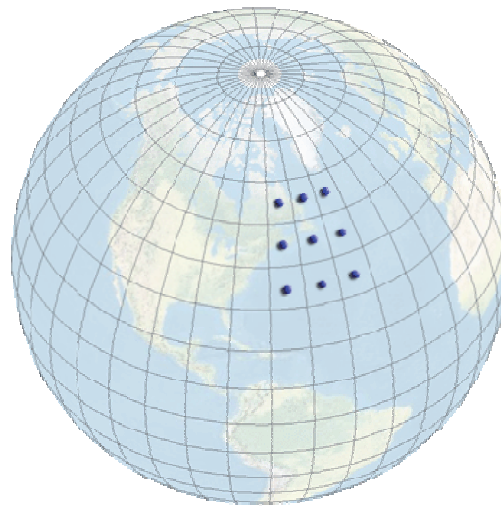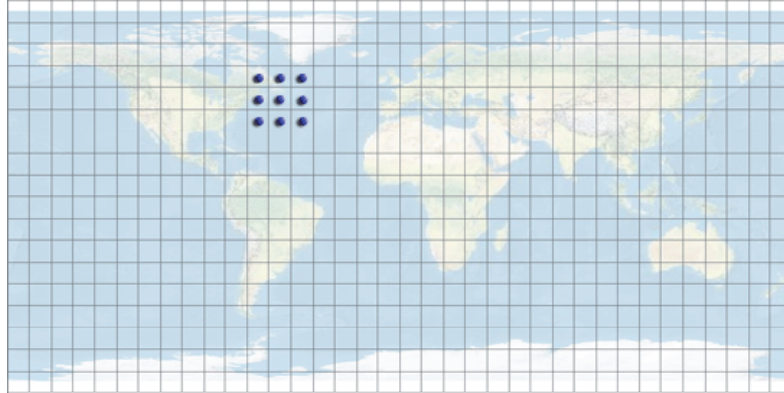
In this case, we cannot treat the cell size as a single constant value. The methods to correct for latitude/longitude data go beyond the scope of this article but are described in Jenness (2011; see especially p. 40 - 49 of the manual at http://www.jennessent.com/downloads/DEM%20Surface%20Tools%20for%20ArcGIS.pdf). For those who are interested, the author's free ArcGIS extension *DEM Surface Tools* will automatically correct for latitude/longitude data (see http://www.jennessent.com/arcgis/surface_area.htm).

Solar Insolation: An alternative to Aspect

Aspect has a well-established history in habitat analysis. It is both easy to measure and a good predictor of certain habitat characteristics. However, in some cases we are really interested in how much direct sunlight hits an area (insolation), which is a function of aspect, slope, nearby topography, landscape reflectivity and atmospheric effects, and as such may be a more important driver of habitat characteristics than aspect alone. In such cases, it may be worthwhile to estimate insolation directly rather than use aspect as a surrogate. There are a few approaches you can take:

ESRI's Solar Radiation Tool

As of version 9.2 of ArcGIS and Spatial Analyst, ESRI has included a tool in the ArcToolbox to calculate solar radiation for specific points on the landscape, or over the entire landscape.

Digital Elevation Model

Solar Radiation (kWh/m²)   Hours of Sun Exposure
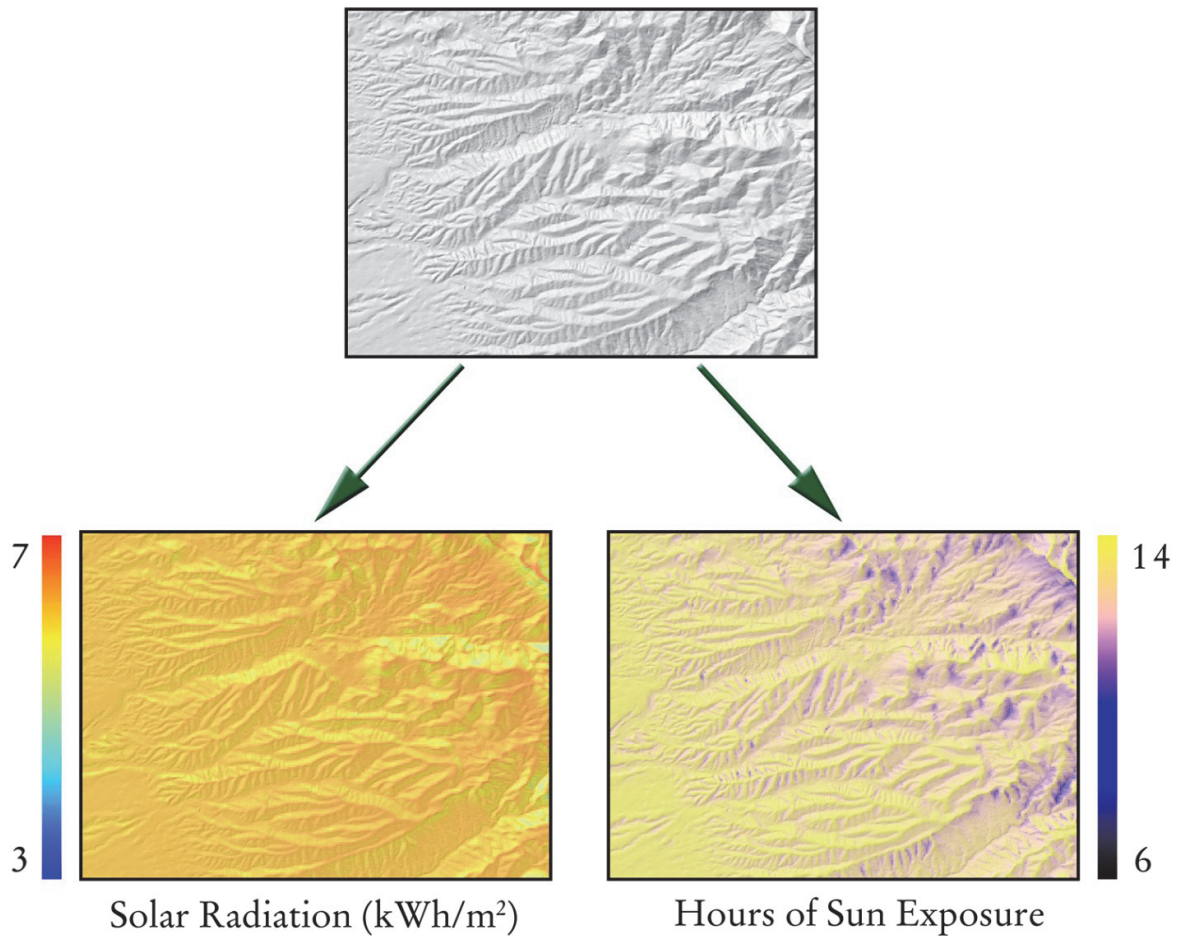
This function incorporates both direct and diffuse radiation and shadows from nearby topography. It includes several useful options, including:

1) Outputs either the amount of energy hitting the ground (in Watts per Hour), or the total number of hours in which the ground is exposed to the sun.

2) Can be calculated for specific dates, seasons or years.

3) Has optional parameters where you can specify the general atmospheric conditions in your area of interest.

4) Has optional parameters where you can specify how intensively it examines the local topography before determining the amount of radiation hitting an area.

In general, this is a wonderful and exciting new tool. I have noticed two minor drawbacks to the tool:

1) It is slow on large grids, and on occasion I have needed to let it run for hours or days.

2) I do not believe that it incorporates reflectivity off the landscape. This would be hard to model, of course, and would depend on exactly how reflective your landscape is (snow reflects very differently from lava rocks, for example).

Despite these two minor drawbacks, I expect that this tool will become very valuable for habitat analysis.

Hillshade

For those who do not have access to the Solar Analyst, you can use the hillshade function available in many GIS packages get a reasonable estimate for the relative amount of sunlight that hits the landscape at a single point in time. Values will typically range from 0 (meaning no sunlight hits the landscape) to 255 (meaning the angle of the landscape is facing the sun directly). Hillshades are difficult to calculate if your DEM is in geographic coordinates, but the author offers a free ArcGIS tool to do this (see http://www.jennessent.com/arcgis/surface_area.htm) which does not require Spatial Analyst.
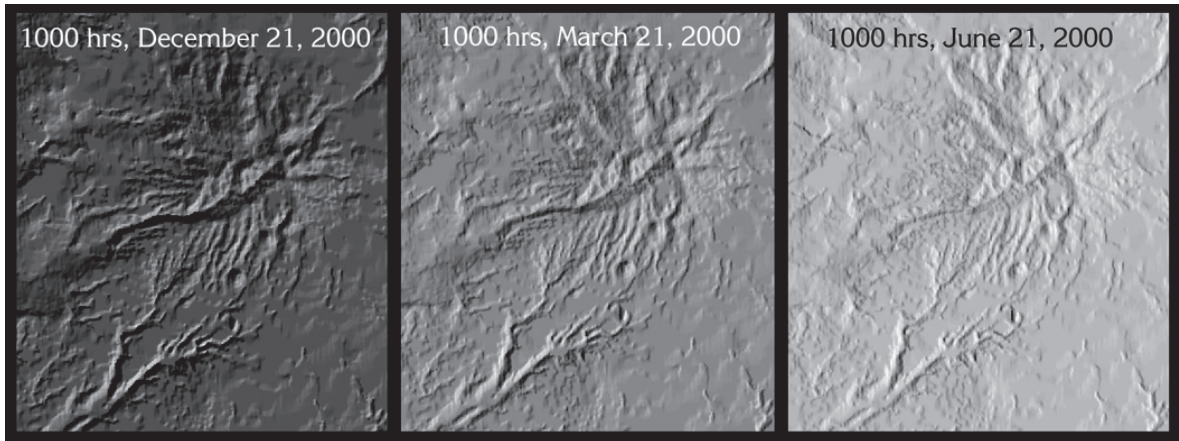
Using the Tools

Convert Shapes to Segments

This function decomposes a complex feature into a set of individual segments, making it easier to analyze the directional characteristics of those segments. For example, a single polyline might represent the entire movement history of an animal during its migration. Because it is a single object in the database, it is difficult to decompose into a form suitable for directional analysis.

This function generates a polyline feature class representing the vertex-to-vertex segments from the features of a polygon or polyline feature class. Each polyline will represent a single straight-line segment. You have the option to generate segments for all features, or only the selected set of features.

This function offers several options to calculate additional data describing each segment, including the starting and ending coordinates, the azimuth of the segment and the reverse azimuth of the segment.

If your data are in latitude/longitude coordinates, then you also have the option to calculate both the starting and ending azimuths of each segment plus the reverse starting and ending azimuths. If this last option is confusing, recall that great circle geodesic curves almost always change their bearing constantly over the length of the curve (see "Azimuths and Geodesic Curves" on p. 16).

Click the **Shapes to Segments** button to open the "Features to Segments" dialog:
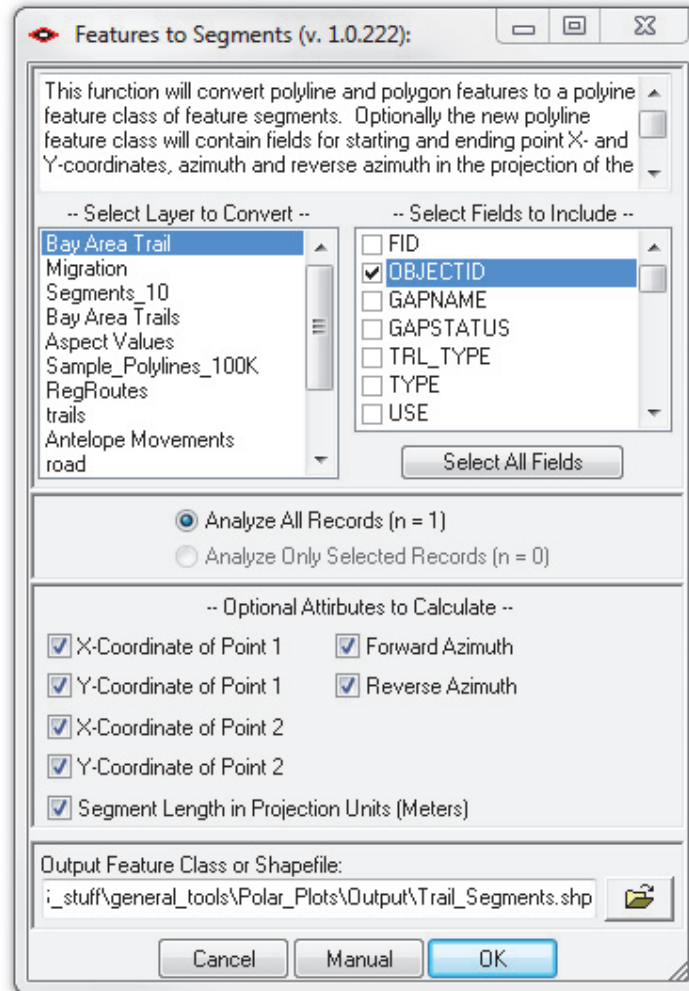
Select the layer to convert, any attribute fields you want to transfer to your new feature class, whether to do all or selected records, any additional attributes describing the geometry of each segment, and where to save your new feature class. Click 'OK' and the tool will create the segment feature class and add it to your ArcMap document.
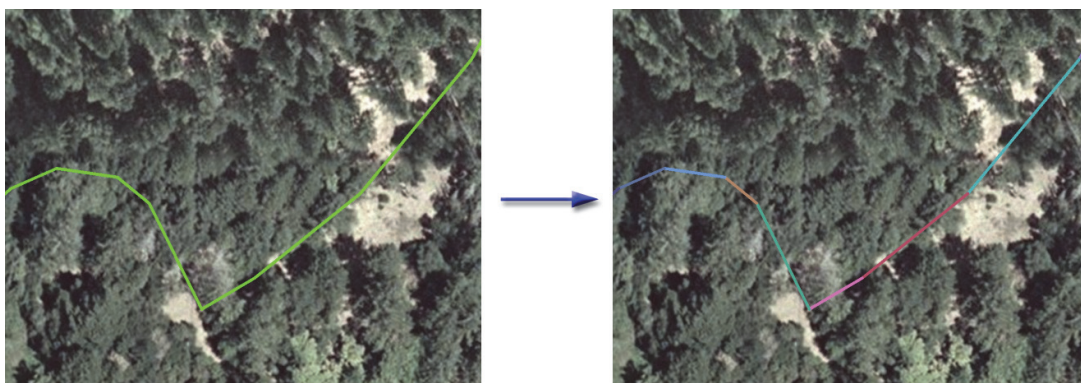
The tool will also generate a report describing what it did.

Report

Shapes to Segments Successful:

Input Parameters:
  ** Input Feature Layer: Bay Area Trail
  ** Convert All Records (n = 1)

  ** Original 'Bay Area Trail' Fields to Include:
     1] Original Field Name = [OBJECTID], Added as [OBJECTID]

  ** Optional Fields Added:
     1) [Start_X]: X-Coordinate of Segment Starting Point
     2) [Start_Y]: Y-Coordinate of Segment Starting Point
     3) [End_X]: X-Coordinate of Segment Ending Point
     4) [End_Y]: Y-Coordinate of Segment Ending Point
     5) [Seg_Length]: Segment Length in Projection Units (Meters)
     6) [Fwd_Azmth]: Segment Azimuth
     7) [Rev_Azmth]: Reverse Segment Azimuth

Output:
  ** D:\arcGIS_stuff\general_tools\Polar_Plots\Output\\Trail_Segments.shp
  ** Spatial Reference: NAD_1983_UTM_Zone_10N
  ** Contains 78 segments

Polar Plots Tool version 1.0.223

             Analysis Began: *Thursday, July 05, 2012 at 2:19:58 PM*
        Analysis Complete: *Thursday, July 05, 2012 at 2:20:22 PM*
              Time Elapsed: *Time Elapsed: 24 seconds...*

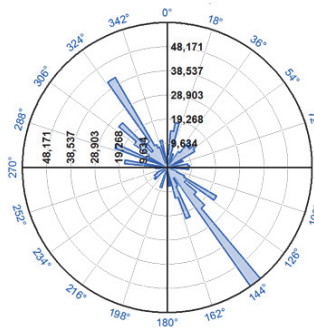Print   Copy to Clipboard   Exit
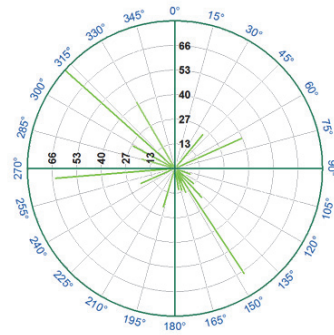
Generate Circular Plots

This tool will generate 6 types of circular plots, with numerous options regarding colors, labels, titles and reference lines.
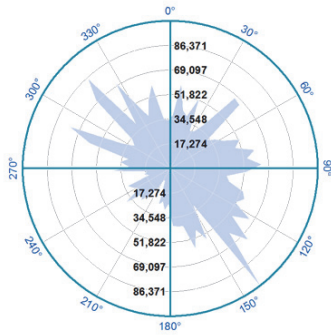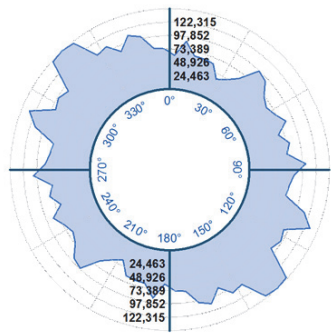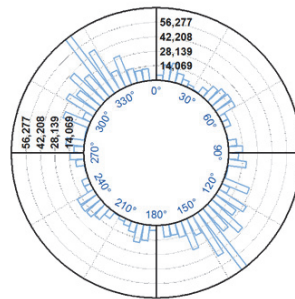

Peaks and Valleys
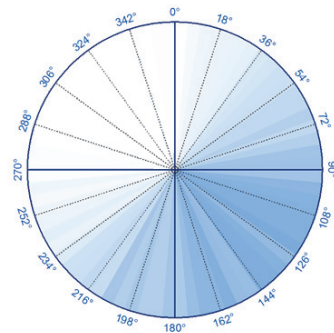
Circular Bins
(aka Rose Plot)

Bearings

External Peaks
and Valleys

External Bins

Shade by Density

The tool will generate the plot as a graphic in the MXD Layout and therefore you must have the layout active when you generate the plot. If you try to generate the plot while the Map view is active, the tool will offer to switch you to the layout view automatically.

The graphic can be copied and pasted to other software packages (Microsoft Word and PowerPoint, for example). However, for best results I recommend exporting the graphic into a standard image file using the standard ArcGIS Export functions.

The graphic will actually be a "Grouped" graphic where each component is a separate graphic. If you create your graphic plot and want to modify any single portion of it, simply:

1) Select the plot graphic using the ArcGIS "Select Elements" tool

2) Right-click on the graphic and choose "Ungroup"

3) Select the individual element you want to edit using the "Select Elements" tool

4) Right-click on it and choose "Properties".

Note: The "Bearings" plot type is only intended for cases with less than a few hundred bearings. This graphic takes a long time to draw and can be miserable to work with if you have more than a few hundred bearings.

Click the **Polar Plots** button ⊕ to open the Polar Plot Parameters dialog:
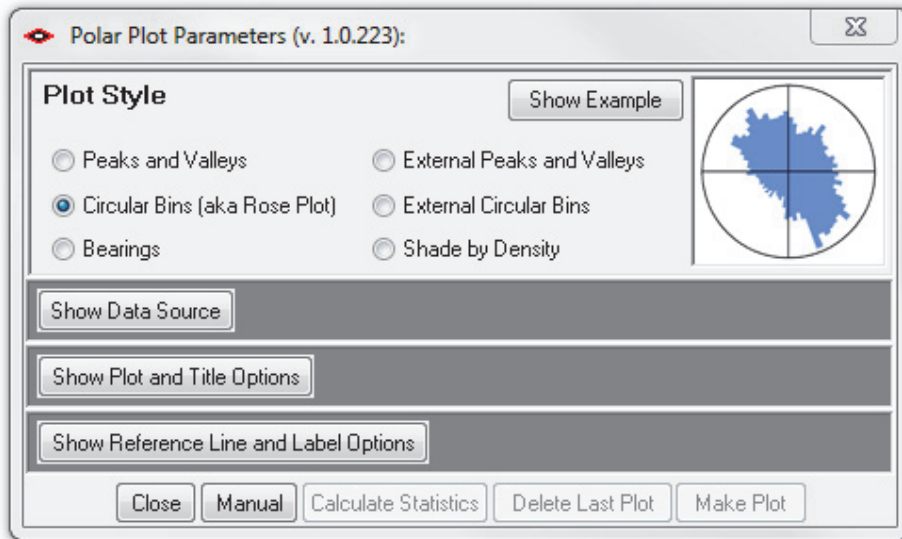
There are a lot of options on this dialog, so they are broken down into subsets and are accessed by clicking the subset button. For example, the "Show Data Source" button open s up the data source options and lets you pick the dataset to plot, plus specify the source for azimuth and weight values.

Plot Style

The "Plot Style" section is always visible and shows the six plot style options available. To see a larger example of any of the plot styles, click the "Show Example" button. This larger example also has most of the plot, title, reference line and label options pointed out:
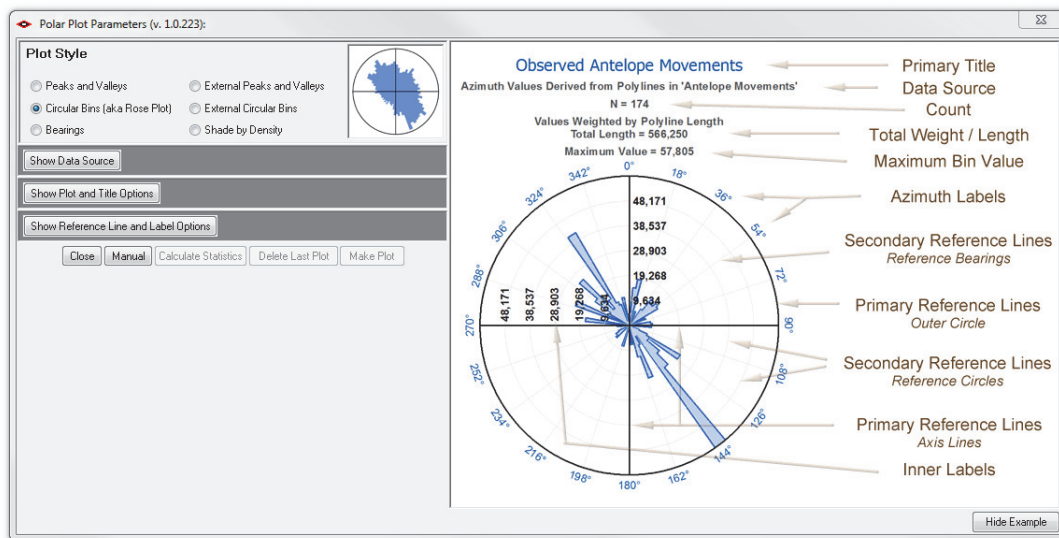
Data Source Options:

Click the "Show Data Source" button to open the Data Source options:
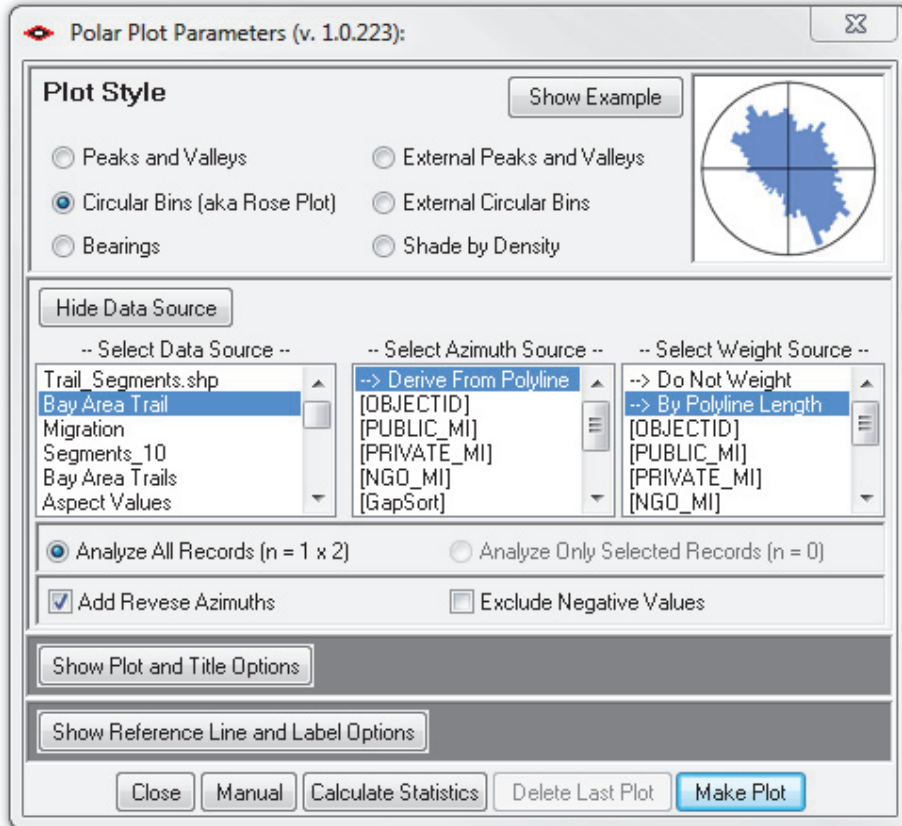
- **Data Source:** The list includes all feature classes and tables in your current focus map. ***Note: This version of the tool does not work with raster datasets.*** If you wish to plot the distribution of a raster dataset (of aspect values, for example), there are two approaches you might try:

  1) Convert the raster to a point feature class, then select the points that lie within your area of interest, and then run this tool on the selected points.

  2) Convert the raster to an Integer-type raster (which has an associated attribute table containing "Value" and "Count" fields), then export the attribute table to a standalone table, and then run the tool on the attribute table. Use the "Value" field as the source of azimuth values, and the "Count" field as the source of weight values.

- **Azimuth Source:** If your data source is a *Table*, *Point Feature Class* or *Polygon Feature Class*, then you must set your source of azimuth values to an attribute field in the attribute table. If your data source is a *Polyline Feature Class*, then you may derive your azimuth values direction from the polyline features. In this case, azimuth is defined as the azimuth from the starting point to the ending point of the polyline. If the polyline is in latitude/longitude coordinates, then this tool uses the starting azimuth value (see "Azimuths and Geodesic Curves" on p. 16 for an explanation of starting vs. ending azimuth values).

- **Weight Source:** If your direction values have different weights (for example, if your data source is a polyline feature class of segments and each segment might be a different length), then you may choose to weight your direction values by some attribute value. If your data source is a polyline feature class, then you also have the option to weight each polyline by the length of that polyline.

- **Analyze All vs. Selected Records:** If any of your records are selected, then you have the option to only analyze that selected set.

- **Add Reverse Azimuths:** If your dataset represents a phenomena in which both directions should be plotted (such as geologic fracture lines on the landscape), then this option will plot both the observed azimuth value for each record plus the azimuth in the reverse direction. In a plot with reversed azimuths, every bin value will be reflected by a bin of the exact same length going in the opposite direction.
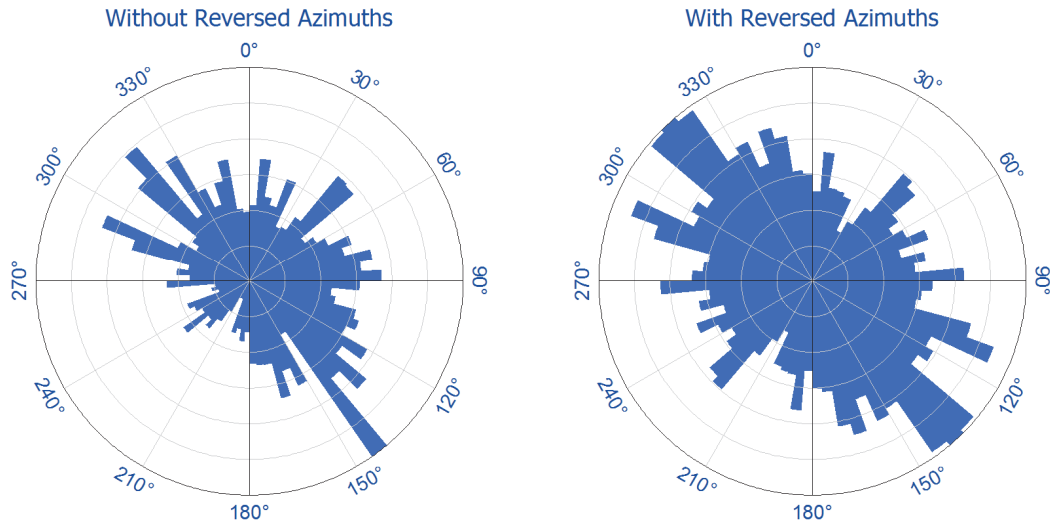
- **Exclude Negative Values:** Depending on your data, negative values may reflect genuine directions (counter-clockwise from north) or they may be a flag that the value is an invalid azimuth. For example, aspect-generating algorithms often produce a -1 value if the raster cell is flat (i.e. if it has no aspect). This option lets you exclude any negative values from the analysis.

Plot and Title Options:

These options allow you to set the symbolization for the plot, plus optionally add and symbolize primary and secondary titles.

Click the "Show Plot and Title Options" button to show the plot color and title options:
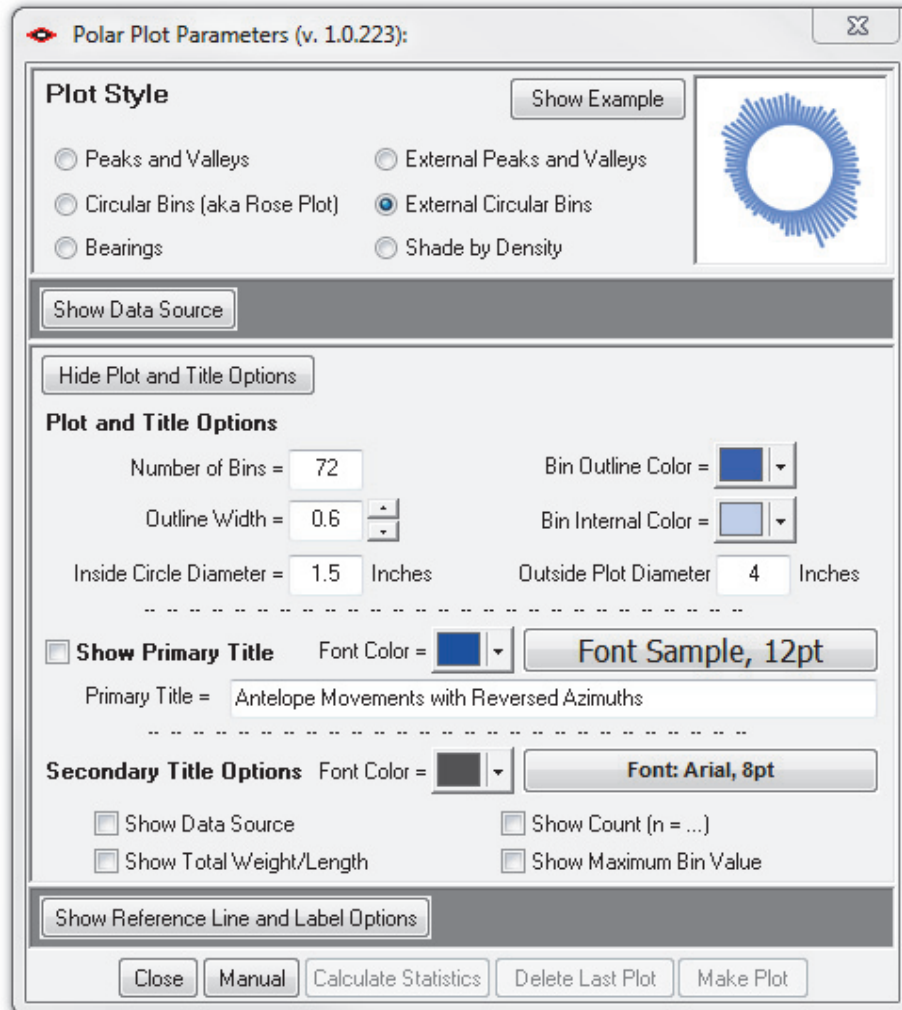
Different plot styles will have different options available. For example, both the "External Peaks and Valleys" and the "External Circular Bins" require *Inside Circle Diameter* and *Outside Plot Diameter* values, while all other plot types only require a single plot diameter value.

- **Number of Bins:** This is the number of bins around the circle. A value of 72 means that each bin will cover 5°. A value of 180 means that each bin will cover 2°.

- **Bin Outline and Internal Colors:** The internal plot of bins has both an internal color and an outline color. Either color can be set to transparent.

- **Bin Outline Width:** In points. If you do not wish to see the outline, you may either set this value to 0 or set the outline color transparent.

- **Bearing Line Color** *(only for "Bearings" plot)*: This is the color of the bearing lines in the plot.

- **Bearing Line Width** *(only for "Bearings" plot)*: The width of the bearing lines in the plot, in points. This value must be ≥ 0.2.

- **Low and High Density Colors** *(only for "Shade by Density" plot)*: The bin with the lowest total value will be colored the "Low Density Color" value and the bin with the highest

value will be colored the "High Density Color". Intermediate bins will be colored according to a color ramp from the low to high color.

- **Plot Diameter** *(only for "Peaks and Valleys", "Circular Bins", "Bearings" and "Shade by Density" plots)*: This is the diameter of the circle, in inches, that contains all the data. It does **not** include any labels outside this circle.

- **Inside Circle Diameter** *(only for "External Peaks and Valleys" and "External Circular Bins" plots)*: This is the diameter of the inner circle, in inches, which forms the base of the plot bins.

- **Outside Plot Diameter** *(only for "External Peaks and Valleys" and "External Circular Bins" plots)*: This is the diameter of the circle, in inches, that contains all the data. It does **not** include any titles outside this circle.
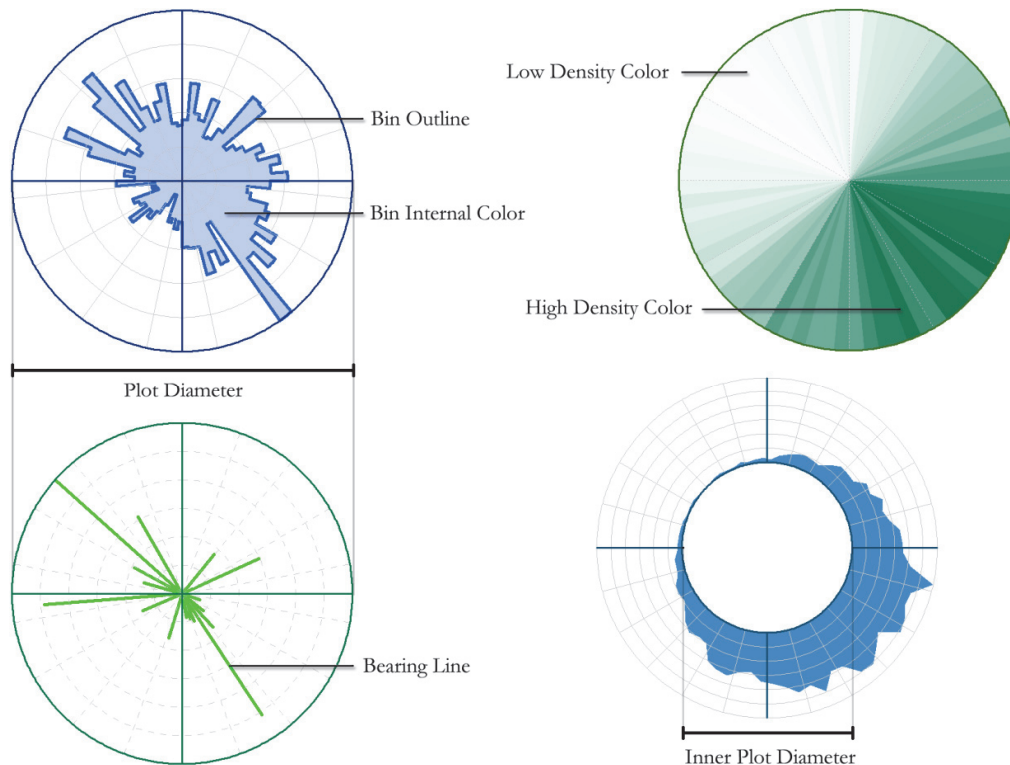
- **Primary Title**: Check this option and specify a title if you wish to add a title to your graphic. You may also specify the font style, size and color.

- **Secondary Titles**: All secondary titles will use the same font style, size and color. You have the option to add any or all of 4 secondary titles:

  o <u>Data Source</u>, including both the dataset and the source of azimuth values.

  o <u>Count of values analyzed</u>. This may be either the total number of records or the number of selected records depending on your choice. If you chose to add reverse azimuth values, then the count will be doubled. If you chose to exclude negative values, then the count will only include the azimuths ≥ 0.

  o <u>Total Weight/Length</u>: If you weighted your azimuth values, then this option will show the weighting source and the total cumulative weight. If you did not weight your azimuth values, then this option will add the line "Values Not Weighted" to your plot.

o   <u>Maximum Bin Value</u> *(this applies to all plot types except for "Shade by Bearing" plots)*: This is the maximum bin value. If you weighted your azimuth values, then this will be the largest cumulative weight in a bin. Otherwise this will be the maximum number of azimuth values in a bin.

Peaks and Valleys Plot Type ——————————— Primary Title

Azimuth Values Derived from Polylines in 'Bay Area Trails' ———— Secondary Title: *Data Source*

N = 3,326 ———— Secondary Title: *Count*

Values Weighted by Polyline Length
Total Length = 3,130,658 ———— Secondary Title: *Total Weight*

Maximum Value = 103,645 ———— Secondary Title: *Maximum Bin Value*
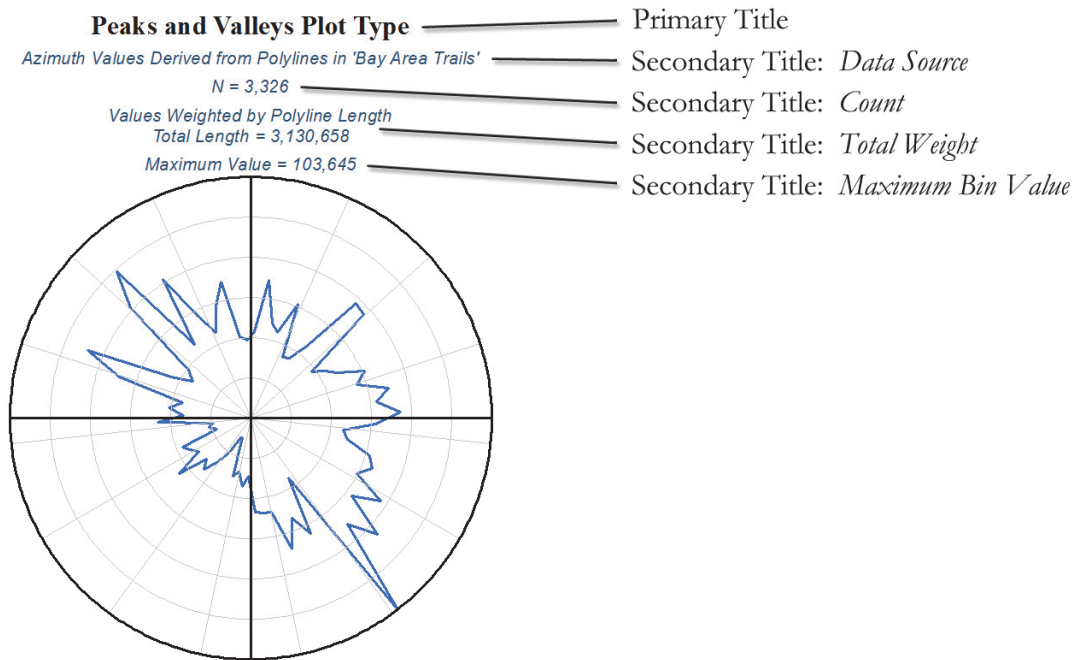
Reference Line and Label Options:

These options allow you to optionally add, symbolize and label reference lines.

Click the "Show Reference Line and Label Options" button to show the various options:
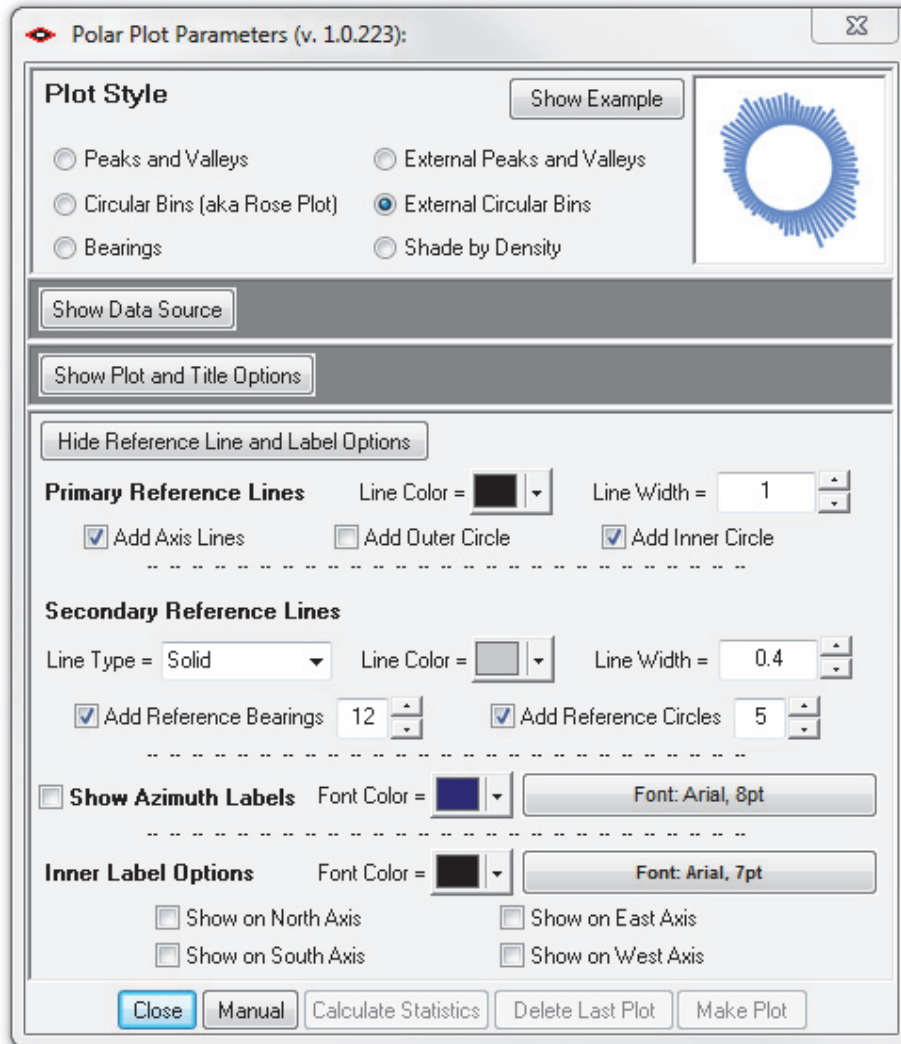
Reference lines are divided into *Primary* and *Secondary* reference lines. These two types can by symbolized differently. For example, you may want to set your primary reference lines in a darker color and with a thicker line than your secondary reference lines. Primary reference lines will always be drawn as a solid line, but secondary reference lines may optionally be drawn as solid lines, dots, dashes or mixed dots and dashes.

Both primary and secondary reference lines are drawn for both azimuths and values. *Azimuth* reference lines extend out from the center as rays, while *Value* reference lines are drawn as progressively-larger circles.

- **Primary Azimuth** reference lines are only drawn at the North, East, South and West directions.

- The **Primary Azimuth Outer Circle** is drawn at the maximum bin value.

- The **Primary Azimuth Inner Circle** is only available in the *External Peaks and Valleys* and the *External Circular Bins* plot types, and is drawn at the base of the bins.

- Any number ≥ 3 **Secondary Reference Line Bearings** may be added. A value of 12 means that secondary reference lines will be drawn every 30°. A value of 15 will produce reference lines every 24°, and a value of 18 will produce lines every 20°.

41

- Any number ≥ 1 **Secondary Reference Circles** may be added. *This option applies to all plot types except for "Shade by Bearing" plots.* This is the number of circles between the 0-level and the maximum bin level. A value of 1 will put a single circle halfway between 0 and the maximum bin level.

- **Azimuth Labels** are the numeric Degree values around the outside of the *Peaks and Valleys, Circular Bins, Bearings* and *Shade by Density* plot types, or around the inside of the *External Peaks and Valleys* and *External Circular Bins* plot types. These azimuth labels will be drawn at all primary and secondary reference azimuths.

- **Inner Labels** show the bin value at all of the secondary reference circle levels. These inner labels may be drawn on any or all of the North, East, South and West axes. *This option applies to all plot types except for "Shade by Bearing" plots.* If no secondary reference circles are drawn, then the *Inner Label* options will be disabled. **Note:** There is no option to add the bin value at the maximum circle level. However, there is an option to add this maximum bin value to the title.
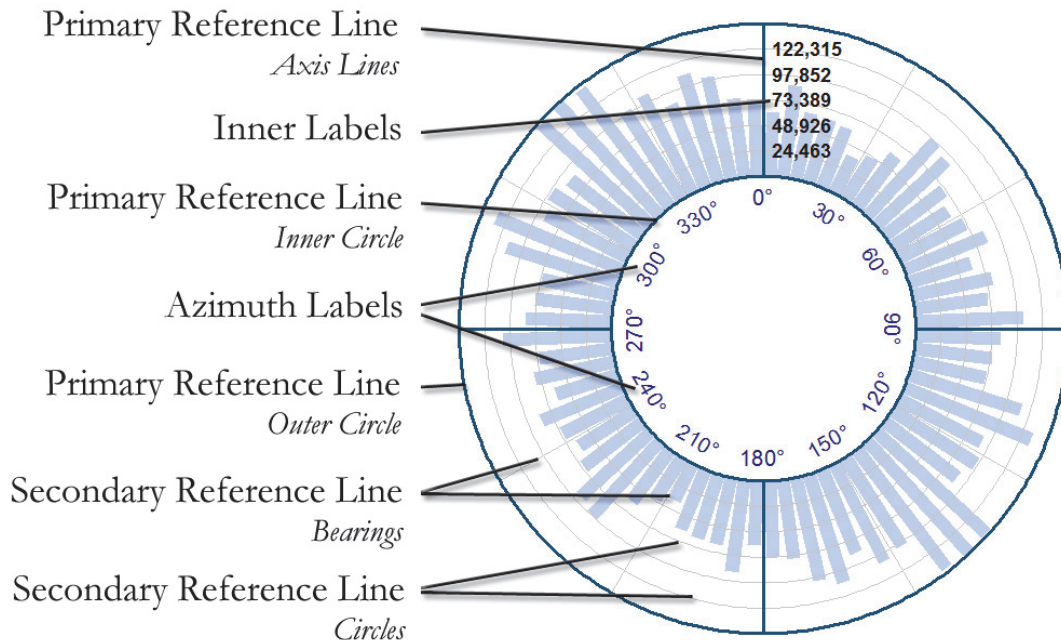
Calculate Descriptive Circular Statistics

This tool calculates the basic set of circular stats describing central tendency and dispersion described on p. 20, including the *Mean Direction, Resultant Length, Mean Resultant Length, Circular Variance, Circular Standard Deviation* (in Radians and Degrees)*, Angular Variance* and *Angular Deviation* (in Radians and Degrees).

These Circular Statistics are available in the same tool that generates the polar plots. Click the **Polar Plots** button ⊕ to open the Polar Plot Parameters dialog:
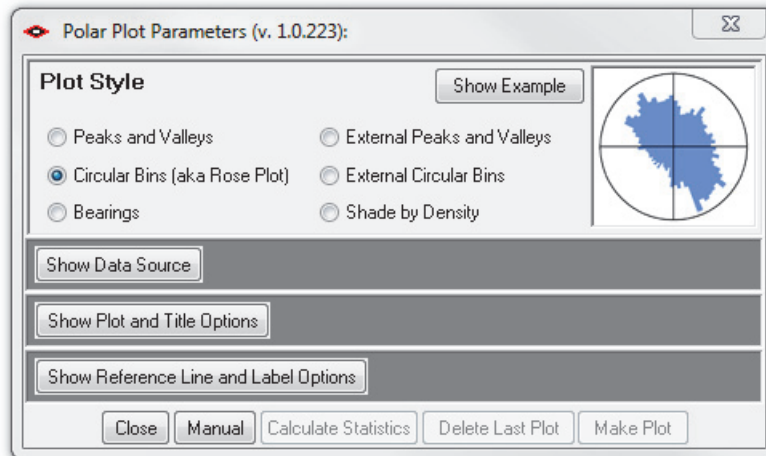
Click the "Show Data Source" button to open the Data Source options section:
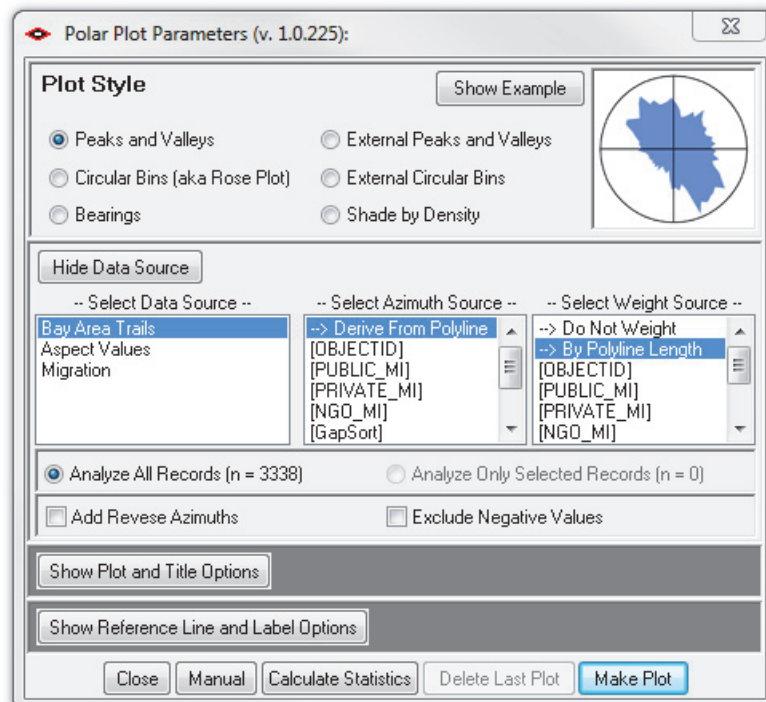
Select the dataset to analyze, the azimuth source and optionally a weighting source, and then click the "Calculate Statistics" button at the bottom of the dialog. The tool will give you the statistics in a separate window:

Report

Circular Descriptive Statistics

Input Parameters:
  ** Dataset = **Bay Area Trails**
  ** Azimuth Source = **Derived from Polylines**
  ** Weighting Option = **By Polyline Length**

Mean Direction = 61.10772 Degrees
Resultant Length = 413546.54646
Mean Resultant Length = 0.1321
Circular Variance = 0.8679   [A,B]
Angular Variance = 1.73581   [C]
Circular Standard Deviation (Radians) = 2.01208 Radians   [A,B]
Circular Standard Deviation (Degrees) = 115.28357 Degrees   [A,B]
Angular Deviation (Radians) = 1.3175 Radians   [C]
Angular Deviation (Degrees) = 75.48724 Degrees   [C]

[A] Fisher, N. I. 1995. Statistical Analysis of Circular Data. Cambridge University Press. p. 32-33]
[B] Mardia, K. V. and P. E. Jupp. Directional Statistics. Wiley Series in Probability and Statistics. John Wiley and Sons, Ltc. p. 18-19]
[C] Batschelet, E. 1981. Circular Statistics in Biology. Academic Press. p. 33-34]

Polar Plots Tool version 1.0.225

Analysis Began: *Friday, July 06, 2012 at 10:59:41 AM*
Analysis Complete: *Friday, July 06, 2012 at 10:59:41 AM*
Time Elapsed: *Time Elapsed: 0 seconds...*

Print   Copy to Clipboard   Exit

Revisions

Version 1.0.225 (July 6, 2012):

- Initial Release

Version 1.0.237 (September 12, 2012):

- Fixed an error in which it would not work with non-Polyline feature classes, instead giving you a message stating that the specified feature class was not a polyline feature class.
- Also updated the Calculate Statistics function so it would tell you how many features or rows it analyzed.

Version 1.0.250 (September 20, 2013):

- Added Von Mises Kappa measure of concentration to statistics.
- Modified analysis count (in plot and in statistics report) to show correct number if any records were excluded from the analysis (negative values, null features, missing azimuth or weight values, etc.).
- A few cosmetic changes.
- Fixed an error that produced the message "Invalid procedure call or argument" at line 1328 of frmLayerID.frm when the "Add Reverse Azimuths" button was clicked when no data sources were available in the map.

Version 1.0.253 (June 23, 2014):

- Modified "Features to Segments" tool to clear up a mysterious bug that caused ArcGIS to shut down completely in some cases. Also modified tool to run faster in general, and especially with two-vertex polylines.

References

Batschelet, Edward. 1981. *Circular statistics in biology*. London; New York: Academic Press.

Beers, Thomas W., Peter E. Dress, and Lee C. Wensel. 1966. "Notes and Observations: Aspect Transformation in Site Productivity Research." *Journal of Forestry* 64 (10): 691–692.

Burrough, Peter A, and Rachael A McDonnell. 1998. *Principles of geographical information systems*. Oxford: Oxford University Press.

Fisher, N. I. 1995. *Statistical analysis of circular data*. Cambridge [England]; New York: Cambridge University Press.

Hodgson, M. 1995. *What Cell Size Does the Computed Slope/aspect Angle Represent?* Vol. 61. Photogrammetric Engineering and Remote Sensing. Bethesda, MD, ETATS-UNIS: American Society for Photogrammetry and Remote Sensing.

Horn, B.K.P. 1981. "Hill Shading and the Reflectance Map." *Proceedings of the IEEE* 69 (1): 14– 47. doi:10.1109/PROC.1981.11918.

Jammalamadaka, S. Rao, and A SenGupta. 2001. *Topics in circular statistics*. River Edge, N.J.: World Scientific. http://site.ebrary.com/id/10255767.

Jenness, Jeff. 2011. *DEM Surface Tools: An ArcGIS Extension for Analyzing Raster Elevation Datasets*. Jenness Enterprises. http://www.jennessent.com/arcgis/surface_area.htm.

Jones, Kevin H. 1998. "A Comparison of Algorithms Used to Compute Hill Slope as a Property of the DEM." *Computers & Geosciences* 24 (4) (May 15): 315–323. doi:10.1016/S0098-3004(98)00032-6.

Mardia, K. V, and P. E Jupp. 2000. *Directional statistics*. New York: Wiley.

Seyfarth, Ernst -August, and Friedrich G. Barth. 1972. "Compound Slit Sense Organs on the Spider Leg: Mechanoreceptors Involved in Kinesthetic Orientation." *Journal of Comparative Physiology A: Neuroethology, Sensory, Neural, and Behavioral Physiology* 78 (2) (June 1): 176–191. doi:10.1007/BF00693611.

Trimble, G. R., and Sidney Weitzman. 1956. "Site Index Studies of Upland Oaks in the Northern Appalachians." *Forest Science* 2 (3): 162–173.

Zar, Jerrold H. 1999. *Biostatistical Analysis*. 4th ed. Pearson Education.

.